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Empirical Testing of Real Option-Pricing Models

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ABSTRACT

This research is the first to examine the empirical predictions of a real option-pricing model using a large sample of market prices. We find empirical support for a model that incorporates the option to wait to develop land. The option model has explanatory power for predicting transactions prices over and above the intrinsic value. Market prices reflect a premium for the option to wait to invest that has a mean value of 6% in our sample. We also estimate implied standard deviations for individual commercial property prices ranging from 18 to 28% per year.

DESPITE EXTENSIVE TESTING OF option-pricing models for financial assets, virtually no research has addressed the empirical implications of option-based valuation models for real assets.[1] This research is the first effort that examines the empirical predictions of a real option-pricing model using a large sample of market prices. Real options that have been valued in the academic literature include capital investments and natural resources, as well as urban land. The model we consider incorporates the option to wait to invest in the valuation of urban land. This paper provides empirical information about the option-based value of land, relative to its intrinsic value and its market price.

Using data on 2700 land transactions in Seattle, we find a mean option (time) premium of 6% of the theoretical land value. This premium ranges from 1% to 30% in various subsamples. We define the "option premium" as the difference between the intrinsic value and the option model value, divided by the option model value.[2] We also find that the option model has explana-

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[1] The exception is the Paddock, Siegel, and Smith (1988) empirical study discussed below. Theoretical real option-pricing models include Titman (1985), Brennan and Schwartz (1985), McDonald and Siegel (1985, 1986), Majd and Pindyck (1987), Morck, Schwartz, and Stangeland (1989), Brennan (1990), Gibson and Schwartz (1990), Triantis and Hodder (1990), and Williams (1991a).

[2] The mean of 6% is an unweighted average across all sample observations.

tory power over and above the intrinsic value for predicting transaction prices. Therefore, to the extent that it is possible to coordinate and time an investment, valuation models should account for the option to wait. We believe that the premia for more speculative properties might be much larger than the values given here.[3]

In addition, we estimate implied standard deviations of individual commercial real estate asset prices in the range of 18 to 28% per year. This result is a contribution in itself, as a lack of repeat sales data for this class of assets makes it difficult to estimate the price variance directly. The implied variances we estimate are equivalent to Black-Scholes implied volatilities from stock options.

Previous research that evaluates the prices obtained from a real option model is limited to Paddock, Siegel, and Smith (1988). Paddock *et al.* develop an option-based model that values offshore petroleum leases as a function of the market price of oil. For 21 tracts, they compare the prices computed from this model both to the government's discounted cash flow model (which uses the same underlying data inputs) and to industry bids (both highest and geometric mean). The Paddock *et al.* and government models give very highly correlated values, and neither comes close to predicting industry bids. The highest industry bid would generally correspond to the market price, providing a comparison between real option values and transaction prices. While these high bids are more than twice either the option-based or government valuations, the mean industry bid is within the range (either above or below depending on the assigned value for gas) of the alternative valuations. As the authors point out, due to a "winner's curse," the high bid may exceed the true expected tract value.

In Section I we discuss our model that prices land, incorporating the option to wait to develop. The option value is a function of the building developed on the site and development costs. In Section II, the marginal prices of each building's characteristics are estimated using hedonic estimation on a separate sample of 3200 developed properties. Using these results, we calculate the value of an optimally scaled building for each of 2700 undeveloped properties. In Section III we evaluate the theoretical land values given by the option-based model relative to the intrinsic values and to market prices. We present conclusions in Section IV.

I. The Model

The model we consider is a fairly general, infinite horizon, continuous time model that in form most closely resembles Williams (1991a), but also tests

[3] The finding of small but consistently positive premia seems reasonable given that Seattle experienced moderate growth during the sample period (1976 to 1979). However, the exact figures obtained for Seattle may not be representative of the overall economy, given the city's dependence on a single industry (aerospace).

the implications of Titman (1985). The principal features of previous optimal timing models are incorporated.

Through ownership of an undeveloped or underdeveloped property, the landholder holds a perpetual option to construct an optimal size building at an optimal time, subject to zoning restrictions. The cost of development is,

$$X = f + q^{\gamma} x_1, \tag{1}$$

where f represents fixed costs, q is the square footage of the building, γ is the cost elasticity of scale, and x_1 is the development cost per square foot. Development costs are assumed to follow a geometric Brownian motion with a constant drift, α_x, and a constant variance, σ_x^2,

$$dX/X = \alpha_x \, dt + \sigma_x \, dz_x. \tag{2}$$

We assume that the price P of the underlying asset, the building, is observable. The implications of this assumption are discussed in Section III.*B*. P is given by $P = q^{\phi}\varepsilon$, where ε is a function of other attributes of the property and ϕ is the price elasticity of scale.[4] The complete formulation and estimation of P are discussed in Section III.*A*. P follows a geometric Brownian motion with constant drift, α_p, and constant variance, σ_p^2,

$$dP/P = (\alpha_p - x_2) \, dt + \sigma_p \, dz_p, \tag{3}$$

where x_2 are payouts to the developed property and $\rho \, dt$ is the constant correlation between dz_x and dz_p. We require that the cost elasticity of scale, γ, exceed the price elasticity of scale, ϕ.[5]

[4] We allow for $\phi < 1$, giving a concave relationship between price and building size in the option model, which is consistent with the relationship estimated in the hedonic function described in Section III.*A*. This concave relationship may exist for several reasons. Colwell (1992) argues that the cost functions are concave because the exterior walls increase less than proportionally to the floorspace. Concavity in the industry offer curves requires, in turn, that the lower envelope of the offer curves, and hence the hedonic function, must be concave. For the building as a whole, the marginal cost curve must intersect the marginal price curve from below in order to obtain an interior solution for the optimal building size. In the market for commercial space, there might be a downward sloping demand curve for a given location, and it is likely that as the building size grows, the prime rentable space decreases as a proportion of the total space (e.g., more interior offices). In the market for residential space doubling an apartment's size does not normally double the rent, since it would still only suit one family and have one kitchen.

[5] Williams' (1991a) model assumes that unit development costs (x_1) and unit cash inflows to the developed property (x_2) are the underlying stochastic variables, both following lognormal processes. The data that we have provides information about building prices, but not about the rents generated by the building. Therefore, we alter the Williams' model with the assumption that the building price and total construction costs, both dependent on scale, are the state variables. Williams solves for price as a linear function of unit cash inflows ($P = \pi q x_2$, π constant), and he assumes that total development cost is a linear function of unit development cost ($X = x_1 q^{\gamma}$). Therefore the two derivations are formally equivalent. However, we are not required to make the assumption, as Williams does, that price is a linear function of scale and therefore that development costs must exhibit decreasing returns to scale (in order to obtain an interior solution to the optimal scale problem). Instead, with our formulation we require only that the scale cost parameter exceed the price elasticity of scale, thus allowing for either increasing or decreasing returns to scale.

We also make the following assumptions. There is a known riskless instantaneous interest rate, i, which is constant through time and equal for borrowers and lenders. Land owners are price takers, giving a partial equilibrium model.[6] The investment is irreversible, i.e., once the investor has built on the property, it no longer has any optimal timing value. βP is the income to the undeveloped or underdeveloped property.[7]

Finally, we assume that there is an equilibrium in the economy in which contingent claims on the pair of processes for the development costs and building price, (X, P) are uniquely priced.[8] We will represent the corresponding pricing operator by taking the expectation of future cash flows under the risk-adjusted probability measure and discounting at the risk-free rate. This is carried out by changing the drifts of X and P, α_x and α_p, respectively, to $\nu_x \equiv \alpha_x - \lambda_x \sigma_x$ and $\nu_p \equiv (\alpha_p - x_2) - \lambda_p \sigma_p$, where λ_x and λ_p are constant parameters representing the excess mean return per unit of standard deviation. We can then express the value of the undeveloped property, $V(X, P)$, as the solution to the fundamental valuation equation:

$$0 = 0.5\sigma_x^2 X^2 V_{xx} + \sigma_{xp} XPV_{xp} + .5\sigma_p^2 P^2 V_{pp} + \nu_x XV_x + \nu_p PV_p - iV + \beta P, \tag{4}$$

subject to the appropriate boundary conditions.

Making a change of variables, $z \equiv P/X$ and $W(z) \equiv V(X, P)/X$, we obtain:

$$0 = 0.5\omega^2 z^2 W'' + (\nu_p - \nu_x) zW' + (\nu_x - i)W + \beta z, \tag{5}$$

where

$$\omega^2 = \sigma_x^2 - 2\rho\sigma_x\sigma_p + \sigma_p^2.$$

To solve this differential equation, we assume that there is a ratio of the building price to development costs, z, at which it is optimal to build. The investor exercises optimally at this "hurdle ratio," z^*, giving the "smooth-pasting" condition. The Appendix provides a more detailed solution and description of these conditions.

[6] The model assumes that an individual's decision to develop has no impact on the market price of buildings. The empirical implications of this assumption are discussed in Section III.*B*. Williams (1991b) models a Nash equilibrium among developers.

[7] The payouts to the undeveloped property are thereby assumed to be proportional to the developed property value.

[8] This assumption is sometimes derived as a consequence of no arbitrage opportunities in an economy in which there exist tradeable securities whose prices are perfectly correlated with P and X. See, e.g., Titman (1985) and Brennan and Schwartz (1985). Given the nature of the underlying processes, we find it more palatable not to explicitly rely on a hedging argument. An equilibrium similar to the one we assume was explicitly derived by Rubinstein (1976) and applied by Milne and Turnbull (1991). An intermediate solution assumes that the component of the risk in (P, X) that is priced in equilibrium can be dynamically spanned by tradeable securities.

The solution is given as follows:

$$V(P, X) = X(Az^{j} + k), \tag{6}$$

where,

$$A = (z^{*} - 1 - k)(z^{*})^{-j},$$
$$z^{*} = j(1 + k)/(j - 1),$$
$$k = \beta z/(i - \nu_x),$$
$$j = \omega^{-2}\Big(.5\omega^2 + \nu_x - \nu_p + \Big[\omega^2\big(.25\omega^2 - \nu_p - \nu_x + 2i\big) + (\nu_x - \nu_p)^2\Big]^{.5}\Big)$$
$$z = P/X$$

The intrinsic value of the option can be found by taking the limit of (6) as the variance ω goes to zero. This result is given by,

$$\begin{aligned} V^{I}(X, P) &= P - X, \qquad z \geq 1 + k \\ V^{I}(X, P) &= \beta P/(i - \nu_x), \qquad z < 1 + k \end{aligned} \tag{7}$$

If the ratio $z \equiv P/X$ exceeds $1 + k$, the landowner will build immediately. Otherwise he will hold the land for the income it generates.[9]

For tractability, the optimal scale or building square footage, q^{*}, is determined by the initial values of P and X, and is therefore the same for both the option-based value, $V(P, X)$, and the intrinsic value, $V^{I}(P, X)$. This assumption understates the value of the option, as we discuss further in Section III.*B*. q^{*} is found by maximizing the value of the undeveloped property, $V(q) = P(q) - X(q) = q^{\phi}\varepsilon - (f + q^{\gamma}x_1)$, over q. The solution is,

$$\begin{aligned} q^{*} &= (\gamma x_1/\varepsilon\phi)^{(\gamma/(\phi-\gamma))} \qquad & q^{*} < \delta \\ q^{*} &= \delta \qquad & q^{*} \geq \delta, \end{aligned} \tag{8}$$

where δ is the maximum size permitted by the zoning regulations.

Our empirical work examines the option-based value given by (6), compared to the intrinsic value (7) and compared to market prices. The building is assumed to be built to the optimal scale in (8), and the optimal time to build is when the ratio of building price to development costs exceeds z^{*}, in (6).

II. The Data

The primary data set consists of a large number of real estate transactions within the city of Seattle.[10] All properties are within the city limits and are zoned for investment purposes: business, commercial, industrial, or low- and

[9] This "hurdle ratio," $1 + k$, corresponds to the ratio z^{*} in the option-based model. It is found by taking the limit of z^{*} as $\omega \to 0$ ($j \to 1$).

[10] The source of the data is the Real Estate Monitor Corporation.

high-density residential.[11] The data cover the second half of 1976 through the end of 1979 and include the characteristics of 3200 transactions of developed properties (developed to a reasonable approximation of the permitted zoning) and 2700 transactions of unimproved land parcels. The data on the developed properties are used in the hedonic estimation of the potential building values. The unimproved parcels represent the real options, the land which the owner has the option to develop.

The cost function is given by (1). These costs are estimated using the Marshall Valuation Service. This service provides indexes of per-square-foot construction costs for various types and qualities of buildings and assigns multipliers for adjusting these unit costs to particular years and localities.[12] Estimation of the cost scale parameter γ is described in Section III.*C*.

III. Empirical Results

A. Estimates of Building Values

Land is valued as an option, for which the underlying asset is the building that potentially would be built on that site. The price of this building is not observable, and thus must be estimated. The method we employ is hedonic estimation. Hedonic theory focuses on markets in which a generic commodity can embody varying amounts of each of a vector of characteristics or attributes Z. A hedonic price function $p(Z)$ specifies how the market price of a commodity varies as these characteristics vary. Rosen (1974) provides a theoretical framework in which $p(Z)$ emerges as the equilibrium price arising from bids and offers of the suppliers and demanders of the good. The distribution of the quantity, as a function of Z, that is supplied and consumed is also endogenously determined.

We separate the sample into years (1976–1977, 1978, and 1979) and into five zoning categories (commercial, business, industrial, low-density residential, and high-density residential), to improve the predictive power of the coefficients.[13] For each subsample, we regress the log price for an improved

[11] The Seattle Zoning Code classifies these zoning categories cumulatively. The lowest (i.e. most inclusive) zoning category is industrial. Commercial includes nonretail business and light manufacturing. The purpose of business zoning is to provide for retail and office uses. The lowest residential use (high rise and mixed use) is not included in our sample because there were few data points and a large amount of heterogeneity. Therefore, what we term high-density residential is actually medium density (midrise). Low-density residential includes duplexes and triplexes.

[12] We calculate the 1977 to 1979 square foot costs for building types according to the purpose of each zoning category (apartment, store, office, warehouse, and industrial building) for an average quality *C* and a good quality *B* building, and chose values at the middle of each range. These costs range from \$23 per square foot to \$34 per square foot. We assume fixed costs of \$10,000.

[13] A Wald test of the restrictions that the parameters are the same across zoning categories is statistically significant in nearly all cases. Therefore, we separated these groups in this first estimation, and also in the tests of the land valuation model.

property on its characteristics,

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b'L_i + d'Q_i + e_i \tag{9}$$

The independent variables included are the log of square footage of the building (q), the log of the lot (LSF), and the height and age of the building. L is a vector of six dummy variables, obtained by combining groups of census tracts in the city. These are abbreviated *n* (north), *w* (west), *ce* (central east), *cw* (central west), *se* (southeast), and *sw* (southwest). Q is a vector of dummy variables representing the quarter in which the property was sold, with the last quarter of each subsample omitted. This functional form is used because its Box-Cox transformation gives the highest log likelihood, lowest standard error, and highest R^2.

From these equations we estimate the coefficients to be used to determine the potential building value on an undeveloped plot of land. The results from each of these regressions are presented in Tables I to V. The fit of the regressions is good for all zoning categories and all years, with R^2 ranging from 80.2 to 95.6. Each of the price elasticities of size is less than one and significantly greater than zero at conventional levels. The elasticity of lot size is fairly large, in most cases greater than 0.5. The elasticity of building size ranges approximately between 0.3 and 0.5. This situation is reversed for low-density residential properties, for which the elasticity of the building size is much higher than that of the lot. The estimates vary across years; the variation might reflect that each data set consists of the differing properties that have sold each year. However, because of the unknown time series properties of this data, it is possible that the standard errors are understated. Therefore, we cannot reject the hypothesis that the true elasticities are constant across time.

For business and commercial properties, the downtown area (central west, *cw*) is the eliminated locational dummy variable. For all other subsamples the north is eliminated due to the lack of data points in the downtown area. From these estimates it is clear that Seattle is not a monocentric city, and that a simple variable measuring the distance from the city center would not capture important locational features. In particular, the area east of downtown (central east, *ce*) is a location that has negative price effects. North and west generally add to value, while central west is not always the most attractive location.[14]

When the coefficient for age is statistically significant, age has a negative impact. For the height levels that make up most of the sample, the effect of height on value is increasing at a decreasing rate, although the coefficients on these variables often are only marginally significant.

[14] Our goal was to estimate coefficient that serve as predictors of marginal prices. We separated the city into as many regions as possible, given the number of data points in each region.

Table I

Hedonic Estimation for Business Properties in Seattle

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b'L_i + d'Q_i + e_i$$

For each property, i, P is the price; q and LSF are the building and lot sizes, respectively, as measured in square feet. Height (HT) is measured in stories and age in years. L and Q are dummy variables representing the location and quarter in which the sale took place. The last quarter of each time period and Central West are omitted variables. N is the sample size. The regression is estimated separately for each of the three time periods.

	1976–1977			1978			1979	
R-squared	0.922			0.843			0.878	
Std. Error	0.312			0.457			0.426	
F-statistic	156.11			67.31			82.86	
N	215			177			164	
Variable	Coeff.	Std. Error		Coeff.	Std. Error		Coeff.	Std. Error
Constant	−0.741	0.463		3.210	0.575		1.152	0.527
log(q)	0.565	0.039		0.495	0.058		0.314	0.057
log(LSF)	0.875	0.063		0.507	0.067		0.935	0.061
HT	0.126	0.053		0.163	0.066		0.255	0.094
HT^2	−0.011	0.006		−0.006	0.004		−0.019	0.011
AGE	−0.003	0.001		−0.005	0.002		−0.003	0.002
Locations								
North	−0.158	0.089		−0.074	0.147		−0.185	0.134
West	−0.101	0.094		0.070	0.158		−0.262	0.148
Central east	−0.355	0.100		−0.434	0.172		−0.276	0.146
Central west	0			0			0	
Southeast	−0.340	0.094		−0.117	0.172		−0.198	0.159
Southwest	−0.415	0.105		−0.511	0.166		−0.332	0.155
Quarters								
76–3	−0.202	0.078	78–1	−0.258	0.097	79–1	−0.190	0.143
76–4	−0.075	0.065	78–2	−0.167	0.098	79–2	−0.136	0.147
77–1	−0.067	0.081	78–3	−0.007	0.112	79–3	−0.120	0.148
77–2	−0.045	0.070						
77–3	−0.039	0.084						

In order to predict the building value that corresponds to a particular plot of land, we need to estimate the size and height of the building. From Section I, the building price is given by $P(q) = q^{\phi}\varepsilon$ where the building size, q, and the price elasticity of scale, ϕ, are the same as in the hedonic equation (9), and ε is a function of other attributes of the property. We assume that the investor develops the optimally sized building, $q = q^*$, as given by equation (8).

We base the height estimates on existing property heights, so that the estimated height, $\widehat{\mathrm{HT}}$, of a given property is equal to the average height for the relevant zoning and location. We then conclude that the estimated value of a building developed on each of the 2700 land transactions is given by,

$$P_i = q_i^{*\phi} LSF_i^{\psi} \exp\left\{c + a_1\widehat{HT}_i + a_2\widehat{HT}_i^2 + b'L_i + d'Q_i + e_i\right\} \qquad (10)$$

Table II

Hedonic Estimation for Commercial Properties in Seattle

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b'L_i + d'Q_i + e_i$$

For each property, i, P is the price; q and LSF are the building and log sizes, respectively, as measured in square feet. Height (HT) is measured in stories and age in years. L and Q are dummy variables representing the location and quarter in which the sale took place. The last quarter of each time period and Central West are omitted variables. N is the sample size. The regression is estimated separately for each of the three time periods.

	1976–1977			1978			1979	
R-squared	0.885			0.856			0.893	
Std. Error	0.336			0.397			0.326	
F-statistic	108.9			54.26			75.05	
N	229			133			131	
Variable	Coeff.	Std. Error		Coeff.	Std. Error		Coeff.	Std. Error
Constant	1.887	0.366		2.854	0.476		0.853	0.469
log(q)	0.419	0.036		0.501	0.064		0.415	0.050
log(LSF)	0.702	0.038		0.513	0.067		0.804	0.061
HT	0.051	0.068		0.151	0.076		0.209	0.059
HT^2	0.002	0.011		−0.004	0.005		−0.014	0.004
AGE	−0.006	0.001		−0.004	0.002		−0.000	0.002
Locations								
North	−0.119	0.090		0.080	0.146		−0.055	0.103
West	0.144	0.085		0.151	0.154		−0.064	0.102
Central east	−0.208	0.100		−0.267	0.160		0.075	0.127
Central west	0			0	0		0	0
Southeast	−0.283	0.134		0.090	0.261		−0.065	0.185
Southwest	−0.325	0.104		−0.199	0.178		−0.285	0.116
Quarters								
76–3	−0.148	0.079	78–1	−0.228	0.092	79–1	−0.092	0.098
76–4	−0.086	0.078	78–2	−0.122	0.105	79–2	−0.026	0.092
77–1	−0.079	0.073	78–3	−0.008	0.104	79–3	0.059	0.094
77–2	−0.084	0.087						
77–3	0.017	0.067						

where the L, LSF, and Q represent the actual location, size, and date sold of each parcel.

B. Observation Errors

There are a number of sources of error in our estimation. Consistent with the assumption of an exogenously given building price process, the estimated building values do not account for a possible depression in prices due to additions to supply. This would tend to overstate the value of the building if the true price process reflects a downward sloping demand curve. The intrinsic value of the land would tend to be overstated and the option premium correspondingly understated. However, the sample consists of many scattered, mostly small, heterogeneous lots in a market which at the time

Table III

Hedonic Estimation for Industrial Properties in Seattle

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b' L_i + d' Q_i + e_i$$

For each property, i, P is the price; q and LSF are the building and lot sizes, respectively, as measured in square feet. Height (HT) is measured in stories and age in years. L and Q are dummy variables representing the location and quarter in which the sale took place. The last quarter of each time period and North are omitted variables. N is the sample size. The regression is estimated separately for each of the three time periods.

	1976–1977			1978			1979	
R-squared	0.824			0.948			0.956	
Std. Error	0.464			0.318			0.252	
F-statistic	22.0			26.8			45.5	
N	75			33			41	
Variable	Coeff.	Std. Error		Coeff.	Std. Error		Coeff.	Std. Error
Constant	2.740	0.907		2.866	0.923		1.989	0.512
log(q)	0.332	0.085		0.373	0.105		0.289	0.058
log(LSF)	0.728	0.082		0.681	0.110		0.789	0.054
HT_2	−0.329	0.422		−0.177	0.283		0.171	0.202
HT^2	0.085	0.108		0.037	0.042		0.013	0.030
AGE	0.000	0.003		−0.004	0.004		0.001	0.003
Locations								
North	0			0			0	
West	−0.224	0.222		−0.079	0.372		−0.230	0.234
Central east	*			−0.144	0.372		−0.651	0.293
Central west	*			0.225	0.422		1.011	0.316
Southeast	−0.491	0.225		−0.102	0.186		−0.330	0.159
Southwest	−0.288	0.149		−0.108	0.176		−0.277	0.147
Quarters								
76–3	−0.590	0.278	78–1	−0.548	0.302	79–1	−0.353	0.189
76–4	−0.460	0.259	78–2	−0.431	0.241	79–2	−0.262	0.178
77–1	−0.412	0.253	78–3	−0.308	0.233	79–3	−0.169	0.195
77–2	−0.283	0.263						
77–3	−0.107	0.258						

* No transactions in this region.

was neither saturated nor underdeveloped. The supply of land available for development was fairly limited. The building on each of these lots probably does not have much impact on the price. Therefore, the potential for substantially overstating the building value due to the assumption of exogeneity is much smaller than for a large tract of similar properties.

In addition, we may tend to overstate the true price because we observe prices only for those undeveloped or developed properties that sell. However, we also assume that the observable characteristics of existing buildings have the same marginal prices as the new buildings, and we only control for the depreciation of the existing stock in a simple way. This assumption would tend to understate the true price. Note that we do not assume that the new building is developed to the maximum density. We assume only that its value

Table IV

Hedonic Estimation for Low-Density Residential Properties in Seattle

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b'L_i + d'Q_i + e_i$$

For each property, i, P is the price; q and LSF are the building and lot sizes, respectively, as measured in square feet. Height (HT) is measured in stories and age in years. L and Q are dummy variables representing the location and quarter in which the sale took place. The last quarter of each time period and North are omitted variables. N is the sample size. The regression is estimated separately for each of the three time periods.

	1977			1978			1979	
R-squared	0.820			0.815			0.856	
Std. Error	0.288			0.282			0.254	
F-Statistic	112.2			112.7			121.6	
N	361			319			259	
Variable	Coeff.	Std. Error		Coeff.	Std. Error		Coeff.	Std. Error
Constant	2.560	0.400		3.611	0.350		0.948	0.306
log(q)	0.800	0.036		0.622	0.374		0.637	0.034
log(LSF)	0.279	0.044		0.378	0.044		0.631	0.042
HT	0.195	0.114		−0.152	0.159		0.022	0.041
HT^2	−0.054	0.031		0.051	0.046		−0.001	0.002
AGE	−0.006	0.001		−0.005	0.001		−0.001	0.001
Locations								
North	0			0			0	
West	0.127	0.047		0.040	0.052		0.046	0.048
Central east	−0.292	0.047		−0.467	0.049		−0.064	0.053
Central west	*			*			*	
Southeast	−0.073	0.044		−0.072	0.051		−0.097	0.056
Southwest	−0.399	0.057		−0.358	0.052		−0.072	0.050
Quarters								
76-3	−0.359	0.060	78-1	−0.333	0.045	79-1	−0.287	0.061
76-4	−0.309	0.061	78-2	−0.213	0.046	79-2	−0.243	0.045
77-1	−0.241	0.048	78-3	−0.117	0.047	79-3	−0.010	0.044
77-2	−0.235	0.048						
77-3	−0.191	0.047						

* No transactions in this region.

is estimated based on values of other buildings in the same zoning classification, which generally are not developed to the maximum density.[15] In sum, the building value estimation introduces several potential biases, the net impact of which is difficult to gauge.

[15] Because the zoning is cumulative, we attempted a search over all possible building types for a given zoning category, e.g., allowing an industrial parcel to be developed commercially. The prices obtained were unreasonably high, indicating that the industrial parcel could not command the same marginal prices as a commercially zoned parcel. The fact that the land was zoned as industrial conveys information about its location and potential. Moreover, the marginal prices we estimate for the industrial parcel are based only on other industrially zoned lots, which are not necessarily developed to the maximum density.

Table V

Hedonic Estimation for High-Density Residential Properties in Seattle

$$\log P_i = c + \phi \log q_i + \psi \log LSF_i + a_1 HT_i + a_2 HT_i^2 + a_3 AGE_i + b'L_i + d'Q_i + e_i$$

For each property, i, P is the price; q and LSF are the building and lot sizes, respectively, as measured in square feet. Height (HT) is measured in stories and age in years. L and Q are dummy variables representing the location and quarter in which the sale took place. The last quarter of each time period and North are omitted variables. N is the sample size. The regression is estimated separately for each of the three time periods.

	1977			1978			1979	
R-squared	0.859			0.848			0.802	
Std. Error	0.379			0.353			0.350	
F-statistic	161.3			119.4			54.8	
N	413			269			175	
Variable	Coeff.	Std. Error		Coeff.	Std. Error		Coeff.	Std. Error
Constant	3.181	0.274		2.637	0.401		2.472	0.492
log(q)	0.595	0.045		0.772	0.050		0.548	0.063
log(LSF)	0.407	0.051		0.315	0.062		0.593	0.071
HT	0.090	0.069		0.149	0.110		0.015	0.122
HT^2	−0.004	0.010		−0.031	0.020		−0.002	0.023
AGE	−0.007	0.001		−0.003	0.001		−0.003	0.001
Locations								
North	0			0			0	
West	0.118	0.058		*			*	
Central east	−0.159	0.053		0.018	0.070		0.021	0.094
Central west	0.156	0.391		−0.178	0.055		−0.080	0.693
Southeast	−0.056	0.073		0.152	0.089		0.007	0.126
Southwest	−0.282	0.073		−0.203	0.094		0.068	0.136
Quarters								
76–1	−0.296	0.069	78–1	−0.249	0.064	79–1	−0.221	0.123
76–2	−0.252	0.069	78–2	−0.189	0.064	79–2	−0.273	0.126
77–1	−0.254	0.068	78–3	−0.001	0.064	79–3	−0.174	0.124
77–2	−0.212	0.058						
77–3	−0.140	0.059						

* No transactions in this region.

The state variable to the option model is the ratio z of the building price to development costs. Both these values are likely to be observed with error. Since z centers into the model in a nonlinear way, the expected value of the land is affected by the estimation error. To be specific, if we assume that the estimation error is normal and multiplicative, i.e., if $\ln \hat{z} = \ln z + \varepsilon$ and ε is distributed $N(0, \sigma_\varepsilon^2)$, then,

$$E[W(\hat{z})] = Az^j \exp(j^2\sigma_\epsilon^2/2) + k > W(z). \tag{11}$$

The presence of this bias means that we are more likely to reject the null hypothesis, and therefore that our tests tend to be more conservative than stated. In addition, in our regressions of theoretical prices on actual prices, the presence of errors-in-variables problems biases the slope coefficient downward and the intercept upward.[16]

We assumed that the building price is observable. For many reasons, including building delay, the value is usually not observable. As was shown in Flesaker (1991), this uncertainty can lead to errors of omission, in which the option is not exercised when it should be, or errors of commission, in which the option is exercised when it should not be, and uniformly lowers the value of the option itself. However, if the developer retains the option to alter the building plans during the building process, the option value may possibly increase.

C. Results from Tests of the Land Valuation Model

In this section we discuss the results of the estimation and specification tests. As with the hedonic estimation, we break the sample into the five data classes organized by zoning category, for which the parameters are assumed constant across each class. We evaluate real option values, (6), relative to market prices and to the intrinsic values, (7).

In order to calculate the model prices, we must make assumptions about several parameters. We assume that the risk-adjusted drift parameters $\nu_p = \nu_x = 0.03$ and the interest rate $i = 0.08$. The model does not appear to be very sensitive to these assumptions. We find, however, that the theoretical values are extremely sensitive to assumptions regarding the values of the development cost scale parameter, γ, and the payout to the underdeveloped property, β. Since we lack information about the true values of these variables, we estimate values that minimize the pricing errors in our sample. We estimate a value for β ranging from 0.3% to 3% of the developed building value. Our priors were that γ should be less than but close to one, giving some economies of scale. The values we estimate range from 0.9 to 1. The prices we calculate are extremely sensitive to γ, primarily through its function in determining the optimal building size. Only a small fraction of the sample was developed to the maximum density.

Because both the intrinsic value and option-based value models depend in the same way on the building value, development costs, and land payout, the estimates of γ and β, the optimal building size q^*, and the height assumptions should affect the option model and intrinsic value equivalently.[17] That is, for positive variance, reasonable changes in these values do not alter the theoretically positive difference between the option model price and intrinsic value.

Table VI shows variance estimates that are "implied" from the real option model. The parameter we estimate is ω^2, given by (5), which is the variance of the developed property value and development costs. The standard errors

[16] See Theil (1971), for example.

[17] In the intrinsic value case, the building is either developed immediately, a function of the factors which affect P and X, or held as income-producing land, a function of P and β.

Table VI

Variance Estimates Implied from the Option Model

We estimate Var(P, X), the variance of developed property value, P, and development costs, X, from the option model (equation (6)), which incorporates the option to wait to develop land. The standard error is of this estimate. The variance and standard deviation of developed property value, P, are calculated assuming a 5% annual standard deviation of development costs and zero covariance.

	n	Var(P, X)	Standard Error	Var(P)	Standard Deviation (P)
Business					
1977	76	0.0369	0.0030	0.0344	0.1855
1978	64	0.0616	0.0053	0.0591	0.2431
1979	48	0.0571	0.0046	0.0546	0.2337
Commercial					
1977	102	0.0503	0.0024	0.0478	0.2186
1978	90	0.0533	0.0032	0.0508	0.2254
1979	73	0.0526	0.0024	0.0501	0.2238
Industrial					
1977	62	0.0525	0.0037	0.0500	0.2236
1978	43	0.0813	0.0038	0.0788	0.2807
1979	25	0.0658	0.0073	0.0633	0.2516
Low-density residential					
1977	490	0.0720	0.0011	0.0695	0.2636
1978	401	0.0577	0.0017	0.0552	0.2348
1979	340	0.0488	0.0016	0.0463	0.2152
High-density residential					
1977	224	0.0475	0.0014	0.0450	0.2121
1978	336	0.0647	0.0031	0.0622	0.2494
1979	360	0.0699	0.0022	0.0674	0.2595

of the estimates are very low. We then present values for the variance and standard deviations of the developed property value only, assuming the $\rho = 0$ and $\sigma_x = 0.05$. We find annual standard deviations ranging from 18.55% to 28.07%, with no significant differences among the different property types.[18] We can reject that the variance is constant, but we do not find any uniform movement up or down in the variance estimates across the years.

Because these figures represent the annual standard deviation of individual properties, based on actual prices rather than on appraisals, it is difficult to find comparable numbers in the literature. The closest we find comes from a recent study by Case and Shiller (1989) of repeat sales; the study reports a 15% annual standard deviation of individual housing prices, from 1970 to 1986, in Atlanta, Chicago, Dallas, and San Francisco-Oakland. Titman and Torous (1989) estimate an implied property value standard deviation of

[18] Clearly, different cities and different sample periods would generate different results. The results obtained in this study provide an indication of how the model fares empirically, but do not purport to be representative.

15.5% using their commercial mortgage-pricing model.[19] There is a fundamental inconsistency in utilizing a model that assumes constant variance to estimate implied variances that are allowed to change. This issue has been addressed in several papers. Preliminary findings of Sheik and Vora (1990) show that, under certain circumstances that allow for changing variance (such as a constant elasticity of variance diffusion process), implied volatilities measure the average volatility of the underlying stocks' returns fairly accurately.

Based on our assumptions and estimates, most properties would not be developed if the investor correctly accounts for the option to wait. For most properties the current ratio of building price to development costs, z, is still less than the optimal development ratio z^*. However, most properties would be developed in the intrinsic value case, where a null variance is assumed. We lack information about the actual development of these properties,[20] and therefore cannot test whether the developer follows an optimal strategy.

In Table VII, we present summary statistics for the option-based model price and the intrinsic value. As expected, the former exceeds the latter in every subsample, and in some cases by a substantial margin. Based on these values, we calculate the option (time) premium as the mean percentage difference bewteen the option model price and the intrinsic value. These premia range from 1% to 30%, with a mean of 6%.[21] In support of the theory, they are consistently positive. There is no reason for the estimated option premium to be constant across the sample, since properties bought for current development should have a premium of zero, while more "speculative" transactions could have premia approaching 100% of the value. We consider that these numbers represent a lower bound on the option premium for land, since our sample consists of urban land during a period of expansion in a city with tight growth controls.[22] Some larger figures appear, especially for industrial properties.[23] When the industrial properties are excluded, the average premium is 5%. As previously discussed, our assumptions and

[19] By comparison, Cox and Rubinstein (1985) report individual stock standard deviations ranging from 17% (for utilities) to 68% (for Winnebago) during the 1980 to 1984 sample period.

[20] From 1977 to 1979 Seattle experienced a steady increase in building permits for all property types, but building activity subsequently dropped off. No trend can be seen in our data which reflects the statistics.

[21] This mean is calculated across all observations, i.e., $\Sigma_{t=1}^{15}(N_t op_t)/N$, where for each subsample i, N_i is the number of observations and op_t is the option premium, and N is the total number of observations (2734).

[22] We would expect higher premia in locations where very little building is currently taking place, indicating that the value in the land is mostly as an option to build far out in the future. The overall expansion in Seattle during this period indicates that many options are "in the money."

[23] The premia for the industrial properties are not driven by just a few outliers. The industrial properties have by far the widest range of sizes and prices compared to the other zoning categories. The land prices and lot sizes are also the largest. Given the small sample size and heterogeneous data, it is quite possible that the building values fitted from the hedonic regressions are less representative of the sample of vacant lots than for the other categories. It is also possible that the industrial properties in 1977 and 1978 do have a larger option premium.

Table VII

Summary Statistics and Option Premia

Average land values given by the option model (equation (6)), which incorporates the option to wait to invest, and the intrinsic value (equation (7)) which does not value this option. The option premium for each parcel is defined as: (option model price − intrinsic value)/option model price. We present the mean option premium for each subsample (not the option premium of the average values). Sample sizes are as given in Table VI.

	Option Model ($)	Intrinsic Value ($)	Option Premium
Business			
1977	30,550	29,090	0.0377
1978	115,092	112,578	0.0222
1979	84,773	74,998	0.0449
Commercial			
1977	144,237	136,844	0.0518
1978	180,221	177,735	0.0095
1979	184,477	171,792	0.0256
Industrial			
1977	146,670	122,124	0.2980
1978	337,196	291,904	0.1757
1979	147,812	140,696	0.0219
Low-density residential			
1977	90,106	84,603	0.0489
1978	47,207	43,968	0.1120
1979	51,148	49,192	0.0117
High-density residential			
1977	58,147	54,148	0.1040
1978	40,981	37,512	0.0586
1979	51,227	48,404	0.0189

imputations should not affect the existence of a premium for the option to wait. However, the standard errors may be large despite the narrow confidence intervals given by the variance estimates.

Finally, we perform several regressions to ascertain the comparative fit and explanatory power of the option-pricing model. In these regressions, errors-in-variables bias the slope coefficient downward and the intercept upward. In Table VIII we regress theoretical prices for both the option and intrinsic value models given in equations (6) and (7), respectively, on actual prices, where all prices are per square foot. Both models perform fairly well as measured by their R^2, although overall we reject the joint hypothesis that the coefficient is one and the constant is zero.

In order to assess the incremental effect of the option to wait, we also run the regressions using the observed land values as the dependent variables, with the intrinsic values and the difference between the option model values and the intrinsic values as independent variables (values per square foot). The latter is a measure of the option premium, in dollar terms. Results are presented in Table IX. The difference between R^2 in Table IX and those in the right side of Table VIII represent the additional explanatory power of the

Table VIII

Regressions of per Square Foot Market Prices on Model Prices

The models are well specified if the coefficients are not significantly different from zero, and the constants are not significantly different from one. Option model: Market Price/SF = $a + b$*Option Model Price/SF + e. Option model price (equation (6)) incorporates the option to wait to invest. Intrinsic value: Market Price/SF = $a + b$*Intrinsic Value/SF + e. Intrinsic value (equation (7)) does not value this option. It equals the option model price for which the variance is zero in the limit. Sample sizes are as given in Table VI.

	Option Model					Intrinsic Value				
	Constant a	Std. Error	Coeff. b	Std. Error	R-square	Constant a	Std. Error	Coeff. b	Std. Error	R-square
Business										
1977	0.7355	0.0957	0.7814	0.0179	0.963	0.6609	0.0798	0.9110	0.0171	0.975
1978	−0.5551	0.2962	1.0913	0.0373	0.932	−0.6102	0.2662	1.1606	0.0354	0.945
1979	0.0919	0.1780	0.9781	0.0296	0.960	1.2966	0.1193	0.9666	0.0234	0.974
Commercial										
1977	0.6387	0.0900	0.8234	0.0166	0.961	1.3372	0.0791	0.7985	0.0163	0.960
1978	−0.8580	0.3470	1.1364	0.0293	0.945	0.1117	0.3533	1.0826	0.0304	0.935
1979	1.8306	0.2857	0.7705	0.0304	0.900	2.1786	0.3027	0.8182	0.0360	0.878
Industrial										
1977	−0.1859	0.1584	1.0786	0.0365	0.936	0.1157	0.1163	1.1502	0.0301	0.960
1978	−0.4262	0.1212	1.0973	0.0196	0.987	0.6801	0.1481	0.9998	0.0255	0.974
1979	2.0889	0.1367	0.6173	0.0264	0.960	2.3146	0.1737	0.5993	0.0349	0.928
Low-density residential										
1977	−1.4781	0.1864	1.1566	0.0211	0.860	1.9440	0.1092	0.9131	0.0080	0.964
1978	−0.3560	0.1079	1.0662	0.0128	0.945	0.1129	0.1103	1.0307	0.0133	0.938
1979	0.6242	0.0727	0.8807	0.0068	0.981	1.3223	0.0656	0.9142	0.0067	0.982
High-density residential										
1977	−0.5059	0.1200	1.1068	0.0146	0.963	−0.3230	0.1494	1.1261	0.0189	0.941
1978	0.8254	0.3344	0.9001	0.0545	0.449	2.3718	0.2671	0.7261	0.0475	0.412
1979	1.1399	0.0818	0.7987	0.0103	0.944	2.6650	0.0762	0.6772	0.0105	0.921

Table IX

Regressions per Square Foot

Market Price of Land Parcel/SF = $a + b$ Intrinsic Value/SF + c(Option Model Value/SF − Intrinsic Value/SF). Option model price (equation (6)) incorporates the option to wait to invest. Intrinsic value, given by intrinsic value (equation (7)) does not value this option. Sample sizes are as given in Table VI.

	Constant a	Std. Error	Coeff. b	Std. Error	Coeff. c	Std. Error	R-square
Business							
1977	0.3412	0.0753	0.9372	0.01361	0.6993	0.09605	0.985
1978	−0.7711	0.2431	1.1721	0.03203	0.3739	0.18748	0.956
1979	1.2424	0.4509	0.9700	0.03601	0.0424	0.34273	0.979
Commercial							
1977	0.4839	0.1730	0.8924	0.02261	0.7835	0.14528	0.969
1978	−1.0618	0.5015	1.1508	0.03611	1.3296	0.42118	0.942
1979	1.4745	0.3358	0.8390	0.03356	0.6439	0.18164	0.898
Industrial							
1977	0.1332	0.0670	1.0705	0.01881	0.5285	0.04791	0.987
1978	0.3627	0.0746	0.9821	0.01211	0.6352	0.05281	0.994
1979	1.5415	0.1651	0.7318	0.03203	0.5581	0.06866	0.960
Low-density residential							
1977	0.5663	0.2016	0.9679	0.01021	0.6537	0.08228	0.968
1978	−0.0838	0.0742	0.9719	0.00929	1.2678	0.057	0.972
1979	−0.2634	0.1327	1.0217	0.0099	1.2962	0.09926	0.988
High-density residential							
1977	0.0782	0.1086	1.0093	0.01544	0.9332	0.06207	0.971
1978	−0.0537	0.5213	0.9939	0.06778	1.5105	0.28259	0.458
1979	0.3955	0.1520	0.8963	0.01573	1.0843	0.06712	0.954

option premium, over and above the intrinsic value. We find evidence of some contribution based on this criteria, but not generally a significant one.

If the option valuation model were a perfect description of land values the constant would equal zero and the other coefficients would equal one. We find that the constant is not far from zero. The intrinsic value has a coefficient that is close to one. We also find that the coefficients for the option premium are uniformly positive and statistically significant in all but one subsample. While in most cases we would reject the hypothesis that the coefficient equals one, in most subsamples it lies between 0.5 and 1.3, supporting the hypothesis that the option valuation model has some explanatory power for prices, over and above the intrinsic value.

IV. Conclusion

This paper provides evidence, based on a large sample of actual real estate transactions, that the real option-pricing model has descriptive value. Market prices reflect a premium for optimal development, which based on our estimates has a mean of 6% of the land value. The basis behind the theory, that the option to wait has value, appears to ring true. To the extent that it is possible to take advantage of the optimal timing option, its value should not be neglected. We also estimate that the annual standard deviation of individual commercial real estate asset values ranges from 18 to 28%, without relying on time series of property prices or appraised values.

These results encourage further research in this field: using the techniques employed here to test other real option applications, including more speculative properties where we would expect the variance and option premium to be higher. An alternative test might also examine the exercise policy of the developer to gauge whether development did actually occur at the optimal point predicted by the option-based model.

Appendix

We wish to solve equation (4). The solution has the form, $W(z) = Az^{j} + k$.

Inserting the values of W, W', and W'' into equation (4), we obtain values for k and j, given in equation (5).

At the hurdle ratio, z^*, the value of the option is its intrinsic value, giving the first boundary condition, $W(z^*) = Az^{*j} + k = z^* - 1$.

The second boundary condition is the smooth pasting condition, which is based on an assumption of rational exercise, $W'(z^*) = jAz^{*j^1} = 1$.

The solutions for z^* and A follow.

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