# Time Series Analysis

Spring 2010
Jin-Lung Lin

**Course:** 1 semester, 3 hours per lecture.
**Lecturing time:** Tuesday, 6:10pm-9:00pm
**Classroom:** A105.
**Office hours:** Tue: 2:00-5:00pm
**Grading:** Homework: 30%, midterm: 30%, term paper: 40%

This course focuses exclusively on Time Series Analysis (TSA) designated for graduate students majoring in economics or finance. Cointegration and financial econometrics are two main topics but with only one-semester, I need to make a quick pass on the former one so that I can devote more time on the second one. I shall start

The course starts with a lecture introducing stochastic process, time series model and statistical package *R*. I then spend 3 lectures covering conventional univariate time analysis, including identification, estimation, diagnostic checking and forecasting of a time series model. Unit root and cointegration econometrics makes the second part. The third and main part comprises univariate ARCH/GARCH, multivariate GARCH models and stochastic volatility models. A brief review of continuous-time econometrics concludes this course.

Similar to any other filed of economics and finance, intuition and creative ideas constitute the flesh and bone of TSA. I am aiming at equipping the students with proper tools for advanced empirical work and lay the foundation for theoretical research in TSA. In additional to econometric theory, I also emphasize computational aspects of these complicated econometric techniques. *R*, is the main statistical packages used in this course. Homework assignments using *R* will be given but there is no programming question in the midterm exam.

**Textbook:**

- Ruey S. Tsay, 2005, **Analysis of Financial Time Series** 2nd edition, New York: John Wiley

**Reference Books:**

- G.E.P. Box, G.M. Jenkins and G.C. Reinsel, **Time Series Analysis: Forecasting and Control,** 3rd edition, Prentice Hall, 1994.

- Clive W.J. Granger **Forecasting Economic Time Series,** 2nd edition Academic Press 1986
- *Johansen, S. (1995) **Likelihood-based inference in cointegrated vector autoregressive models**, Oxford: Oxford University Press
- Helmut Lutkepohl, Introduction to multiple Time Series Analysis, 2nd edition, Springer-Verlag 1993
- Aris Spanos, **Statistical Foundations of Econometric Modelling**, Cambridge University Press, 1986.

## Topics

1. Introduction to Stochastic Process, Time series and *R* ( 1 lecture)
2. ARIMA modelling ( 2 lectures)
3. Theory of Forecasting ( 1 lecture)
4. VAR and Impulse response analysis (1 lecture)
5. Introduction to unit root and cointegration (2 lectures)
6. Univariate GARCH (2 lectures)
7. Multivariate GARCH (2 lectures)
8. Stochastic Volatility Model (2 lectures)
9. Continuous-time time series models ( 3 lectures)

# 1 Introduction to stochastic processes

Spanos chap 8

- definition
- memory and heterogeneity
  - stationary
  - Martingale

  - Markov

- Brownian motion
  - derivation
  - nowhere differentiability
  - role in stochastic integral

- ARIMA processes

## 2 Univariate ARIMA modelling

Granger & Newbold chap 3

- Autocorrelation, partial autocorrelation function, inverse autocorrelation function
- Wold representation theorem
- Random walk model
- General ARIMA model
- Variance stabilization transformation
- Model identification
  - using ACF & PACF
  - using AIC, BIC, & SC criterion
    C.Z. Wei (1992), "On predictive least squares," **The Annals of Statistics, 22**, pp 1-42.
- Estimation
  - method of moment
  - maximum likelihood method
  - nonlinear estimation
- diagnostic checking
  Box-Pierce Q-statistics Box, G.E. & D.A. Pierce (1970) ,"Distribution of residual autocorrelations in autoregressive-integrated moving average time series models," **Journal of American Statistician Association, 52**, 181-192.

# 3 Theory of Forecasting

Granger and Newbold chap 4

- loss function
- optimal forecast when the parameters are known
- optimal forecast when the parameters are estimated
  C.Z. Wei(1985), "Adaptive predictions by least squares predictors in stochastic regressions with applications to time series," **The Annals of Statistics, 15**, 1667-1782.
- optimal multi-step forecast
  Tiao, G. and D.Xu (1993), "Robustness of maximum likelihood estimators for multi-step prediction–the exponential smoothing case," **Biometrika, 80**, 623-641.
- partial least squares, principal components, ridge regression
  Lin, J and R. Tsay, 2005, "Comparisons of Forecasting Methods with Many Predictors," manuscript.

# 4 Functional central limit theorem and continuous mapping theorem

Billingsley chaps 1,2 P.C.B. Phillips (1987), "Time series regression with a unit root," **Econometrica, 55**, 277-301.

- functional central limit theorem
  - for iid
  - for mixing processes
- continuous mapping theorem

# 5 Unit Root Econometrics

- A little history about unit root and cointegration
  - Yule (1927) periodicity of sunspot numbers
  - Box-Jenkins (1976)

  - Dickey-Fuller (1979)
  - Granger (1981)
  - Nelson and Plosser (1982)
  - Granger and Weiss (1982)
  - Engle and Granger (1987)
  - Phillips (1987)
  - Johansen (1988)

- Why does unit root matter ?
  - properties of I(1) vs. I(0)
  - unit root does exist in economic data

- testing unit root
  D.A. Dickey and W. Fuller(1979), "Distribution of the estimators for autoregressive time series with a unit root," **Journal of American Statistical Association, 74**, 427-31.
  D.A. Dickey and W. Fuller (1981), "Likelihood ratio statistics for autoregressive time series with a unit root," **Econometrica 49**, 1057-72.
  N.H. Chan and C.Z. Wei (1988), "Limiting distribution of least-square estimates of unstable processes," **The Annals of Statistics, 16,'**, 367-401.
  - testing unit root without intercept
  - testing unit root with intercept
  - testing unit root with autocorrelated residual
    * Augmented Dickey-Fuller Test (Fuller chap 8)
    * Phillips-Perron Test
  - testing multiple unit roots
  - near unit root

- spurious regression
  C.W.J. Granger and P Newbold (1974), "Spurious regressions in econometrics, ", **Journal of Econometrics, 2**, 111-120.
  P.C.B. Phillips(1986), "Understanding spurious regressions in econometrics," **Journal of Econometrics, 33, 311-34**.

- Statistical inference with integrated regressor

# 6 Cointegration

R. Engle and C.W.J. Granger(1987), "Co-integration and error correction: representation, estimation and testing," **Econometrica, 55**, 251-76.
S. Johansen(1988), "Statistical Analysis of Cointegration vectors," **Journal of Economic Dynamics and Control, 12**, 231-254.
S. Johansen (1991), "Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, " **Econometrica, 59**, 1551-1580.
S. Johansen and K. Juselius (1990), "Maximum likelihood estimation and inference on cointegration- with applications to the demand for money, " **Oxford Bulletin of Economics and Statistics, 54,** 169-210.
S. Johansen (1992), "Cointegration in partial systems and efficiency of single equation analysis," **Journal of Econometrics**, 52, 389-402.

- definition of cointegration
- economic and statistical meaning of cointegration
- cointegration and error correction model
- estimating and testing cointegration in bivariate system
- estimating and testing cointegration in multivariate system
- empirical examples of cointegration using Taiwan data
- estimating common trend
- threshold cointegration

# 7 VAR and Impulse Response

- VAR
- Impulse response function for stationary series
- Impulse response function for nonstationary processes
- Impulse response function and causality
- Lutkepohk (1991), chap.3
- Bernanke, B. S. (1986), "Alternative explanations of the money-income correlation," *Carnegie-Rochester Conference Series on Public Policy* , 25, 49-100.

- Blanchard, O. J. and D. Quah (1989) "The dynamic effects of aggregate supply and demand disturbance, " *American Economic Review*, 77, 655-673.
- Phillips, P.C.B. (1998) "Impulse Response and Forecast Error Variance Asymptotics in Nonstationary VAR's," *Journal of Econometrics*, 83 21-56.

# 8 ARCH, ARCH-M, GARCH

R. Engle (1982),"Autoregressive conditional heteroscedasticity with estimates of the UK inflation," **Econometrica, 50**, 987-1008.
T. Bollerslev (1986), "Generalized autoregressive conditional heteroscedasticity," **Journal of Econometrics, 31**, 307-327.
R. Engle, T. Ito and W.L. Lin (1987), "Meteor showers or heat waves?"heteroscedastic intra daily volatility in the foreign market," **Econometrica, 58**, 525-542.

- ARCH
- GARCH
- GARCH-M
- metro-shower

# 9 Multivariate GARCH Models

- Reparameterizations
- VEC, GBEKK, CCC, DCC
- Leverage effects in MGARCH models
- Estimation
- Diagnostic checking
- Applications
- Bauwens,L., S. Laurent and J. V. K. Rombouts (2006), "Multivariate GARCH Mmodels: A survey," **Journal of Applied Econometrics**, 21: 79V109.
- R. Engle (2002) "Dynamic conditional correlationXa simple class of multivariate GARCH models," **Journal of Business and Economic Statistics** 20: 339V350.

- Tse YK, Tsui AKC (2002) "A multivariate GARCH model with time-varying correlations," **Journal of Business and Economic Statistics** 20: 351V362.

# 10 Stochastic volatility models

- SV vs. GARCH
- Estimating SV model
- SV and option pricing
- SVpack in *OX*
- Ghysels, E., Harvey, A.C., Renault, E., 1996. Stochastic volatility, in: Maddala, G., Rao, C.R., Handbook of Statistics, Vol 14. Elsevier Sciences, Amsterdam.
- Liesenfeld, and J-F Richard, 2004, "Classical and Bayesian Analysis of Univariate and Multivariate Stochastic Volatility Models," Department of Economics, Christian-Albrecht-Universitat, Ohlshausenstr. 40-60, 24118 Kiel, Germany Department of Economics, Economics Working Paper, 2004-12.

# 11 Continuous-time econometrics

- Some continuous-time stochastic processes
  - Wiener processes
  - Generalized Wiener processes
  - Ito processes
- Ito's lemma
- Distribution of stock prices and log returns
- Black-Scholes differential equations
- An extension of Ito lemma
- Stochastic integral
- Jump-diffusion models

- estimation of continuous-time models
- Tsay(2005), chap 6.

## Softwares

*R*: freely available at
http://www.r-project.org
*Task view: Empirical finance*
http://cran.r-project.org/src/contrib/Views/Finance.html

- *R packages*
- *urca*: Unit root and cointegration analysis
- *arima, forecasting*:classical time series analysis and forecasting
- *fSeries, fMultivar*: GARCH, and more
- *dse, vars*" multivariate time series analysis

The *Rmetrics* bundle comprised of the fArma, fAsianOptions, fAssets, fBasics, fBonds, fCalendar, fCopulae, fEcofin, fExoticOptions, fExtremes, fGarch, fImport, fMultivar, fNonlinear, fOptions, fPortfolio, fRegression, fSeries, fTrading, fUnitRoots and fUtilities packages contains a very large number of relevant functions for different aspect of empirical and computational finance.