

# Time Series Analysis

## Homework # 1

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1. Define the following terms and give an example for each term where possible:
  - time series
  - stationary process
  - strong mixing process
  - ergodic process
  - martingale process
  - Markov process
  - Brownian motion
  - Kolmogorov consistency theorem
2. Define and explain the following terms:
  - field or algebra
  - sigma field or sigma algebra
  - random variable
  - measurable
  - Borel sets, Borel functions
  - probability space
3. Define ACF(autocorrelation function), PACF(partial autocorrelation function) and ESACF(extended autocorrelation function). Find the ACF and PACF for the following series: processes:
  - $y_t = 0.8y_{t-1} + \epsilon_t$
  - $y_t = \epsilon_t + 0.8\epsilon_{t-1}$
  - $y_t = 0.8y_{t-1} + \epsilon_t + 0.8\epsilon_{t-1}$

4. Explain how to identify a time series model using ACF, PACF and ESACF.
5. Derive the formula for FPE. Explain how to select models using FPE.
6. Derive the formula for AIC. Explain how to select model using AIC.
7. Let  $y_t$  be generated by

$$y_t = \beta x_t + \epsilon_t \quad (1)$$

$$\epsilon_t = \alpha \epsilon_{t-1} + \eta_t \quad (2)$$

where  $|\alpha| < 1$ ,  $x_t$  is non-stochastic regressor and  $\eta_t$  is white noise. Denote  $\hat{\beta}$  be the least square estimate of  $\beta$ .

- Is  $\hat{\beta}$  consistent estimate for  $\beta$ ?
- Is  $\hat{\beta}$  efficient estimate for  $\beta$ ?

Prove or justify your answers.

8. Let  $y_t$  be generated by an ARMA(p,q) as:

$$y_t = a_1 y_{t-1} + \dots + a_p y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q}$$

- (a) Explain why is the OLS of  $y_t$  on  $y_{t-1}, \dots, y_{t-p}$  is inconsistent?
- (b) Let  $p = q = 1$ , derive the method of moment estimators for  $a_1, \theta_1$ . Derive the MLE for  $a_i, i = 1, \dots, p, \theta_j, j = 1, \dots, q$ .
- (c) Give an asymptotic theorem for the MLE above. Be specific about the covariance matrix