

Time Series Analysis:

Exercise 3

due on May 4, 2010

1 Functional central limit theorem (10%)

Let y_t be generated by

$$y_t = (1 - \beta)(\mu_0 + \mu_1 t) + \beta y_{t-1} + \epsilon_t, \quad t = 1, 2, \dots$$

where ϵ_t is iid with mean 0 and variance σ^2 . Under the hypothesis of $\beta = 1$, derive the limiting distribution of

$$\frac{1}{T^2} \sum_{t=1}^T y_{t-1}^2$$

2 Unit Root Test (15%)

Let y_t be generated by

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \text{for } t > 0 \tag{1}$$

where $y_0 = k$, ϵ_t be iid with mean 0 and variance σ^2 .

- (a) What is the limiting distribution of t-statistics for $\rho - \rho_0$ under the null hypothesis that $\rho = \rho_0$, where $|\rho_0| < 1$.
- (b) What is the limiting distribution of t-statistics for $\rho - 1$ under the null hypothesis that $\rho = 1$.
- (c) As above, what is the consequence of using the t-test for $H_0 : \rho = 1$? Be specific.
- (d) Explain in your own words why $\hat{\rho}$ converges to true ρ with different rates when there is unit root or not. However, for t-statistics, no different normalizing factor is needed.
- (e) Now let ϵ_t follow an AR(p) process

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \dots + \alpha_p \epsilon_{t-p} + \xi_t$$

where ξ_t is iid with mean 0 and variance σ_ξ^2 . Now how do you test $H_0 : \rho = 1$? Be specific.

3 Cointegration Analysis (15%)

Let the $p \times 1$ vector be generated by

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \epsilon_t \quad (2)$$

where α, β are two $p \times r$ matrices, $0 \leq r \leq p$, and $\Pi = \alpha\beta'$.

- (a) What are the conditions for X_t to be either $I(1)$ or $I(0)$? That is, X_t has no explosive root and is not $I(2)$.
- (b) Write down the MA representation for ΔX_t . Be specific about the linkage between its long-run impact matrix, C , and α, β .
- (c) Use the Granger Representation Theorem above to describe how the rank of Π determine the long-run property of X_t .
- (d) Explain in your own words the meaning of *long-run equilibrium* and why cointegration implies long-run equilibrium. Give at least one example.