

Time Series Analysis:

Exercise 2

due on April 20, 2010

1. Explain the procedures, simulate and graph a time series of 100 observations and from each of the following models:

- (a) White noise:

$$y_t = \epsilon_t$$

- (b) AR(1):

$$y_t = 0.8y_{t-1} + \epsilon_t$$

- (c) MA(1):

$$y_t = 0.8\epsilon_{t-1} + \epsilon_t$$

- (d) ARMA(1,1)

$$y_t = 0.8y_{t-1} + \epsilon_t - 0.8\epsilon_{t-1}$$

- (e) Random walk

$$y_t = y_{t-1} + \epsilon_t, \quad y_0 = 0$$

- (f) $(1 - B)X_t = (1 - .6B)\epsilon_{t-1}$

- (g) $(1 - B)X_t = 5 + (1 - .6B)\epsilon_{t-1}$

- (h) $(1 - 0.9B)(1 - B)X_t = \epsilon_{t-1}$

- (i) $(1 - 0.9B)(1 - B)X_t = (1 - .5B)\epsilon_{t-1}$

where ϵ_{t-1} is *iid* normal with mean 0 and variance 4.

2. Checking the asymptotic theory

Let y_t be an AR(1): $y_t = \beta y_{t-1} + \epsilon_t$, $\beta = 0.8$, $\sigma^2 = 4$ as in previous question. The asymptotic theory suggests that the OLS estimate $\hat{\beta}$ of the AR coefficient follows an asymptotic normal distribution:

$$\hat{\beta} = \frac{\sum_{t=2}^T (y_t y_{t-1})}{\sum_{t=2}^T y_{t-1}^2} \quad (1)$$

$$\sqrt{T}(\hat{\beta} - \beta) \rightarrow N(0, \sigma_\beta^2) \quad (2)$$

What value should σ_β^2 be? How do you use simulation to check the distribution results? Give your program and results using *R*. Set the replication number be 10000.

3. As is in question (1), let the numbers of observations be 100, 500, 10000 respectively. Generate each series for 10000 times. For each replication, compute the minimal AIC and BIC for $p = 1, \dots, 10$, $q = 1, \dots, 10$ and then compare the percentage of AIC and BIC for choosing the right models, over-estimate the order and underestimate the order.