

Econometric Analysis

Homework 4

Given: Nov 29, 2007

Due: Dec. 13, 2007

Jin-Lung Lin

1 Linear Statistical Models

A linear statistical model relating a random response Y to a set of independent variables $x_1, x_2, \dots, x_k$ is of the form

$$\begin{aligned}y_i &= \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_k x_{ik} + \epsilon_i, \quad i = 1, 2, \dots, n \\Y &= X\beta + \epsilon\end{aligned}$$

where $Y = (y_1, y_2, \dots, y_n)'$, $X = (x_1, x_2, \dots, x_k)$, $x_i = (x_{i1}, x_{i2}, \dots, x_{ni})'$, $i = 1, \dots, k$ and

$$\begin{aligned}E(\epsilon_i) &= 0, \quad \forall i \\V(\epsilon_i) &= \sigma^2, \quad \forall i \\E(\epsilon_i \epsilon_j) &= 0, \quad \forall i \neq j \\E(x_{ij} \epsilon_j) &= 0, \quad i = 1, 2, \dots, n, j = 1, \dots, k\end{aligned}$$

Answer the following questions.

1. Derive the least squares estimate for β .
2. Derive $\hat{Y}, \hat{\epsilon}$
3. Show that $\hat{Y}, \hat{\epsilon}$ are orthogonal to each other.
4. Use the result above to explain the argument of decomposing Y into the sum of two components. That is, $Y = \text{projection} + \text{residual}$.
5. Does this decomposition hold if constant term is not included in the model?
6. Show that $Y'Y = \hat{Y}'\hat{Y} + \hat{\epsilon}'\hat{\epsilon}$
7. Use the result above to derive R^2 .
8. Derive the change in R^2 when a variable is added to a regression.
9. Use the result above to show that R^2 never decreases when a new variable is added to a regression.

10. Show the formula for $\bar{R}^2$.
11. Find $E\hat{\beta}$ and show that it is unbiased.
12. Find $V(\hat{\beta}|X)$

2 Partitioned regression and partial regression

Let $y = X\beta + \epsilon = X_1\beta_1 + X_2\beta_2 + \epsilon$.

1. Write down the normal equations.
2. Derive the least squares estimates for β_1, β_2 .
3. Show that If $X_2'X_1 = 0$, then $\hat{\beta}_2$ is the same as regressing y on X_2 alone.
4. Prove Frisch-Waugh-Lovell Theorem as stated in page 28 of the textbook.
5. Use the result above to explain the meaning of β_1, β_2 in your own words.

3 Exercises in the textbook

Do exercises 3,4,6,11,13 in chap 3 of the textbook.

4 Application: The US Gasoline Market

Use the US gasoline data to estimate the income and price elasticities for gasoline and an estimate of the elasticity of demand with respect to the prices of new and used cars. Specifically, fit the following model:

$$\ln(G/pop) = \beta_0 + \beta_1 \ln(Income/pop) + \beta_2 price_G + \beta_3 P_{newcars} + \beta_4 P_{usedcars} + \epsilon$$

where the notation is explained in the readme file.

1. Find $\hat{\beta}_i, i = 0, 1, 2, 3, 4$
2. Find the standard error for $\hat{\beta}_i, i = 0, 1, 2, 3, 4$
3. Perform the analysis of variance as in page 35 in the textbook.
4. Explain the empirical results.