Econometric Analysis
Homework 2

Given: Oct 26, 2007
Due: October 1, 2007

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1. Find the probabilities and quantiles $U_\alpha, (P(Y < U_\alpha) = \alpha)$ of the following questions.
   (a) Y is uniformly distributed between 0 and 2. Find $P(0.5 < Y < 1.5)$ and $U_{0.1}$.
   (b) Y is distributed as $\chi^2(5)$. Find $P(Y < 5)$ and $U_{0.9}$.
   (c) Y is distributed as Poisson with mean 5. Find $P(Y > 5)$ and $U_{0.05}$
   (d) Y is distributed as Normal with mean -5 and variance 2. Find $P(-3 < Y \leq 3)$ and $U_{0.99}$.

2. If Y has distribution function

$$F(y) = \begin{cases} 0, y \leq 0 \\ y/8, 0 < y < 2 \\ y^2/16, 2 \leq y < 4 \\ 1, y \geq 4 \end{cases}$$

   find the mean and variance of Y.

3. If a point is randomly located in an interval (a,b), and if Y denotes the location of the point, then Y is assumed to have a uniform distribution over (a,b). A plant efficiency expert randomly selects a location along a 500-foot assembly line from which to observe the work habits of the workers on the line. What is the probability that the point she selects
   (a) is within 25 feet of the end of the line?
   (b) is within 25 feet of the beginning of the line?
   (c) is closer to the beginning of the line than to the end of the line?

4. One method of arriving at economic forecasts is to use a consensus approach. A forecast is obtained from each of a large number of analysts; the average of these individual forecasts is the consensus forecast. Suppose that the individual 1996 January prime-interest-rate forecasts of all economic analysts are approximately normally distributed with 7% and standard deviation 2.6 $\%$. If a single analyst is randomly selected from among this group, what is the probability that the analyst's forecast of prime interest rate will

   (a) exceed 11%?

   (b) be less than 9%?

5. Scores on an examination are assumed to be normally distributed with mean 78 and variance 36.

   (a) What is the probability that a person taking the examination scores higher than 72?

   (b) Suppose that students scoring in the top 10 $\%$ of the distribution are to receive an A grade. What is the minimum score a student must achieve to earn an A grade?

   (c) What must be the cutoff point for passing the examination if the examiner wants only the top 28.1 $\%$ of all scores to be passing?

   (d) Approximately what proportion of students have scores 5 or more points above the score that cuts off the lowest 25 $\%$?

   (e) If it is known that a student's score exceeds 72, what is the probability that his or her score exceeds 84.

6. The magnitude of earthquakes recorded in a region of North America can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will

   (a) exceed 3.0 on the Richter scale

   (b) fall between 2.0 and 3.0 on the Richter scale

7. Let $(Y_1, Y_2)'$ be bivariate normal variables with mean $(\mu_1, \mu_2)'$ and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

   (a) Write down the joint density function of (Y_1, Y_2).

(b) Write down the joint moment generating function of (Y_1, Y_2).

(c) Prove that the marginal distribution of Y_1 is normal with mean μ_1 and variance σ_1^2.

(d) Find the conditional distribution of Y_1 given that $Y_2 = y_2$.

(e) Use the results in (4) to show that $E(Y_1) = E[E(Y_1|Y_2)]$.

(f) Let α_1, α_2 be any two constants not equal to zeros. Show that $\alpha_1 Y_1 + \alpha_2 Y_2$ is normal and find its mean and variance.

(g) Let $\rho = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$ be the correlation between Y_1 and Y_2. Prove that $-1 \leq \rho \leq 1$.

(h) Let U be uniformly distributed on $(0, 1)$. Find the transformation $G(U)$ such that $G(U)$ is normally distributed with mean 0 and variance 1.