Econometric Analysis

Homework 3

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1 Least squares regression

Let the regression model be $X_i = \alpha_0 + \alpha_1 Y_i + v_i$, where v_i are iid's with mean 0 and variance σ^2.

1. Derive the least square estimates of $\alpha_0, \alpha_1, \sigma^2$, denoted as $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\sigma}^2$.
2. Assume that Y is fixed regressor. Show that $\hat{\alpha}_1$ can be expressed as a weighted averages of X_i and find the weights.
3. Find $E(\hat{\alpha}_1)$ and $var(\hat{\alpha}_1)$.

2 Estimating the slope (15%)

Let y_t be generated by:

$$y_t = \alpha + \beta x_t + \epsilon_t,\ \ t = 1, \cdots, T \tag{1}$$

where ϵ_t is *iid* with mean 0 and variance σ^2, $\alpha = -1, \beta = 2$, and x_t is fixed regressor with values given in each case below.

- Case 1:
  $x_t = (-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5)', \sigma^2 = 1, T = 11$
- Case 2:
  $x_t = (-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5)', \sigma^2 = 2, T = 11$
- Case 3:
  $x_t = (-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10)', \sigma^2 = 1, T = 11$
- Case 4:
  $x_t = (-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10)', \sigma^2 = 2, T = 11$

- Case 5:
  $x_t = (-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0)', \sigma^2 = 1, T = 11$

- Case 6:
  $x_t = (-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)'$,
  $\sigma^2 = 1, T = 21$.

Let $\hat{\beta}^{(i)}, i = 1, \cdots, 6$ be the least square estimate of β for the i-th case. Answer the following questions.

1. Write down the general formula of $\hat{\beta}$ in term of x_t, y_t as in eq. (1). (2)
2. Rank the precision of $\hat{\beta}^{(i)}, i = 1, \cdots, 6$ in each case. Order from the most precise estimate to the least precise one. Explain your results. No partial credit is given to answer without explanation. (10)
3. Let $|x_t| \leq 10$, $t = 1, \cdots, 20$, and the values of x_t is under your control. What values of x_t you should give to maximize the precision of $\hat{\beta}$?. Explain your answer. Note that ϵ_t, $t = 1, \cdots, 20$ is drawn randomly from $iid\, N(0, 1)$. (4)

3 Partitioned regression and partial regression

Let $y = X\beta + \epsilon = X_1\beta_1 + X_2\beta_2 + \epsilon$.

1. Write down the normal equations.
2. Derive the least squares estimates for β_1, β_2.
3. Show that If $X_2'X_1 = 0$, then $\hat{\beta}_2$ is the same as regressing y on X_2 alone.
4. Prove Frisch-Waugh-Lovell Theorem as stated in page 28.
5. Use the result above to explain the meaning of β_1, β_2 in your own words.

4 Application:The US Gasoline Market

Use the US gasoline data to estimate the income and price elasticities for gasoline and an estimate of the elasticity of demand with respect to the prices of new and used cars. Specifically, fit the following model:

$$ln(G/pop) = \beta_0 + \beta_1 ln(Income/pop) + \beta_2 price_G + \beta_3 P_{newcars} + \beta_4 P_{usedcars} + \epsilon$$

where the notation is explained in the readme file. Let y, a $n \times 1$ vector, denote the dependent variance and x, a $n \times 5$ matrix, denote the explanatory variables. Answer the following questions.

1. Find $X'X, (X'X)^{-1}, X'y$.
2. Find $\hat{\beta}_i, i = 0, 1, 2, 3, 4$
3. Find the standard error for $\hat{\beta}_i, i = 0, 1, 2, 3, 4$
4. Perform the analysis of variance as in page 35 in the textbook.
5. Explain the empirical results.

5 Hypothesis testing

A multiple regression of y on a constant, x_1 and x_2 produces the following results.

$$\hat{y} = 4 + 0.4x_1 + 0.9x_2, R^2 = 8/60, e'e = 520, n = 29, X'X = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{bmatrix}$$

1. Test the hypothesis that two slopes sum to 1.
2. Test the hypothesis that slope on x_1 is 0 by running the restricted regression and comparing the two sums of squared deviations.