Econometric Analysis

Homework 2

Given: Nov. 5, 2008

Due: Nov. 18, 2008

Jin-Lung Lin

1 Basic concepts

Define and explain the following terms.

1. skewness of a random variable
2. Conditional variance
3. law of iterated expectation
4. converge in probability
5. p-value

2 Matrix (10%)

Let

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

Determine

1. A - 2D
2. rank of AD
3. $|D|$
4. A^{-1}
5. eigen values of D

3 Evaluating probabilities

1. Let (X, Y) be standard bivariate normal random variables. What is $P(X = 0)$?

2. Z be normal with mean 1 and variance 4. Evaluate $P(-4.16 < Z < 4.92)$. (Note that $4.16 = 2.58 * 2 - 1, 4.92 = 1.96 * 2 + 1$).

3. Let the accidents Y per week occurring at the intersection follows a Poisson distribution with mean 5. Find the probability that there are less than 5 incidents occurring next month. Writing down the formula is good enough and numerical calculation is not required.

4. For the Taiwan lottery, one selects six regular and one special numbers (without replacement) from numbers 1 to 49. Matching all six regular numbers wins the first prize while matching any 5 of 6 regular numbers and the special number wins the second prize. Assume that the winning numbers are randomly drawn. Compute the probability of any lottery ticket to win the first and second prize respectively.

5. For the World Series, two teams, say Yankee and Red Sox, play a series of games until one wins four games. Assume that the probability that Yankee wins a game is .55. Assume that all games are independently played. Compute the probability that the series ends at 6 games (4 wins and 2 losses).

4 Unconditional and Conditional

Let $Z, \epsilon_1, \epsilon_2$ be standard normal independent of each other. Let

$$\begin{aligned} X &= 2Z + \epsilon_1 \\ Y &= -2Z + \epsilon_2 \end{aligned}$$

Answer the following questions:

1. Find $Cov(X, Y)$. Are X, Y correlated or uncorrelated?

2. Find $Cov(X, Y|Z)$. Conditional upon Z, are X, Y correlated or uncorrelated? Even if you can not prove your claim, make an educated guess and explain your answer as clearly as possible.

5 Estimation

Let $Y_1, \ldots, Y_n$ denote a random sample from a population with mean μ and variance σ^2. Consider the following these estimators for μ:

$$\hat{\mu}_1 = \frac{1}{2}(Y_1 + Y_2), \quad \hat{\mu}_2 = \frac{1}{4}Y_1 + \frac{Y_2 + \ldots + Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n, \quad \hat{\mu}_3 = \bar{Y}$$

a. Show that each of three estimators is unbiased.

b. Find the efficiency of $\hat{\mu_3}$ relative to $\hat{\mu_2}$ and $\hat{\mu_1}$, respectively.

6 Normal Random variables (15%)

Let (Y_1, Y_2) be bivariate normal variables with mean (1,-1) and covariance matrix

$$\Sigma = \begin{bmatrix} 1 & 1 \\ 1 & 4 \end{bmatrix}$$

1. Write down the joint density function of (Y_1, Y_2).
2. Find the conditional distribution of Y_1 given that $Y_2 = 1$.
3. Prove that $E(Y_2) = E[E(Y_2|Y_1)]$.

7 Distribution Theory

The random variables y,x,z have a multivariate normal distribution with mean vector $\mu = [1, 2, 4]$ and covariance matrix

$$\Sigma = \begin{bmatrix} 6 & 1 & 3 \\ 3 & 5 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

Compute the slope and intercept in the conditional mean function $E[y|x]$. Compute the two slopes and the intercept in the conditional mean function $E[y|x, z]$. Is the slope on x the same in the two functions? Explain. Compute the conditional variance $Var[y|x]$. Compute the squared correlation between y and x. Compute the squared correlation between y and $E[y|x]$. Compute the squared correlation between y and $E[y|x, z]$.