Introduction to Mathematical Statistics

Homework #1

Due on Oct. 16, 2013
Jin-Lung Lin

There is extra bonus of 10 points for answering question #6 but the maximum point remains to be 100.

1 Basic concepts

Define and explain the following terms

1. Random variable
2. Conditional distribution $Y|X$ where Y, X are two random variables
3. Bayes' Rule
4. Consistent estimator
5. p-value

2 Normal random variables

1. Give the density function of a normal random variable with mean -1 and variance 2.
2. Give the joint density function of a bivariate normal random variable $(Y_1, Y_2)'$ with mean $(1, -1)'$, variances and covariance $\sigma_1^2 = 1, \sigma_2^2 = 4, \sigma_{12} = 1$.
3. Following the question above, give the conditional density function for $Y_2|Y_1 = 2$.

3 Probability distribution functions

Find the probabilities and quantizes $U_\alpha, (P(Y < U_\alpha) = \alpha)$ of the following questions.

1. Let X be a standard normal random variable. What are $P(X = 0)$ and $P(X \in Q)$ where Q denotes the set of all rational numbers? Explain your answer.
2. Y is uniformly distributed between 0 and 2. Find $P(0.5 < Y < 1.5)$ and $U_{0.1}$.
3. Y is distributed as Normal with mean -5 and variance 2. Find $P(-3 < Y \leq 3)$ and $U_{0.99}$.
4. X be normal distributed with mean 1 and variance 4. Evaluate $P((X-1)^2 < 4 \times 5.411894)$ and $P((X-1)^2 < 1)$ respectively.

5. One method of arriving at economic forecasts is to use a consensus approach. A forecast is obtained from each of a large number of analysts; the average of these individual forecasts is the consensus forecast. Suppose that the individual 1996 January prime-interest-rate forecasts of all economic analysts are approximately normally distributed with 7$\%$ and standard deviation 2.6 $\%$. If a single analyst is randomly selected from among this group, what is the probability that the analyst's forecast of prime interest rate will exceed 11$\%$?

4 Simulation experiments

Draw 1000 random numbers from each of the following distributions. Compute the sample mean, sample variance, and compare them with the true mean and true variance. Finally, draw the histogram of the samples along with the theoretical distribution.

1. Binomial distribution with n=100, p=0.2.
2. Geometric with p=0.1
3. Negative binomial distribution with n=100, r=5, p=0.1.
4. Hypergeometric distribution with n=10, N=20,r=5.
5. Poisson distribution with n=100, $\lambda = 4$.
6. Uniform distribution with $\theta_1 = 0, \theta_2 = 1$.
7. Normal with $\mu = 10, \sigma^2 = 2$
8. Gamma distribution with $\alpha = 1, \beta = 3$
9. Chi-square with $\nu = 10$
10. Beta distribution with $\alpha = 3, \beta = 5$.

5 More Simulation experiments

Let y be normally distributed random variable with mean -1, and variance 2.

1. Simulate $y_i, i = 1, \cdots, n$ for $n = 1000$.
2. Find sample mean of $y_1, \cdots, y_n$.
3. Repeat steps 1 and 2 for 2000 times and find the histogram of $\sqrt{n}(\bar{y}_j - \mu)/(\hat{\sigma}_{y_j})$ where $n = 1000, \bar{y}_j, \hat{\sigma}_{y_j}$ denote the sample mean and standard deviation of the j-th replication respectively and μ is the true mean.