# Lecture Notes on Event Study Analysis

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This lecture is largely based upon chap 4 of J. C. Campbell, A. W. Lo and A. C. MacKinlay (1997), *The Econometrics of Financial Markets* (CLM), New Jersey: Princeton University Press.

## 1 Introduction

### Three Basic assumptions

1. market is efficient,
2. the event was unanticipated
3. there were no confounding effects during the event window

### Typical events under investigations

- mergers and acquisitions, earnings announcements
- issues of new debt or equity
- announcement of macroeconomic variables such as trade deficit
- stock splits

### Seven steps of an event study proposed by CLM

1. Event definition
   Event time (frequency); How much time is needed for the financial market to digest the information?

   often 2 days, event day and the day after to capture the closing effect. Define event window.

2. Selection criteria
   Defining the data. What firms are included in the sample?

3. normal and abnormal returns

   $$\epsilon_{it}^{*} = R_{it} - E[R_{it}|X_t]$$

   where $E[R_{it}|X_t]$ is the normal return, expected return if the event did not happen and $X_t$ is the conditioning variables. $\epsilon_{it}^{*}$, the abnormal return, is the difference between actual return and normal return. There are two common return models: *constant-mean-return model* where $X_t$ is a constant and *market model* where $X_t$ is the market return.

4. Estimation procedure
   Estimation window; often 120 days prior to the event window. OLS, two-step LS, etc.

5. Testing procedures
   Defining null and alternative hypotheses, aggregating the abnormal returns

6. Empirical results
   Presenting the empirical and diagnostic checking results

7. Interpretation and conclusions
   Interpretation and distinguishing competing explanations

# 2 An example

An example from CLM
Investigate the influence of quarterly earnings for 30 firms in the D-J Index from January 1988-December 1993. The five years of data for thirty firms provide a total sample of 600 announcements. For each firm, three pieces of information are complied: the date of the announcement, the actual announced earnings, and a measure of expected earnings. A classification of good news, no news, and bad news is defined using a +/-2.5level. CLM note that of the 600 announcements 189 are good news, 173 2 are no news and the remaining 238 are bad news. With the announcements categorized, a sampling interval, event window and estimation window that will be used to analyze the behavior of the firmss equity returns. A 41 day event window is specified as follows: 20 are pre-event days, the event day and the 20 post-event days. The estimation window is 250 trading days prior to the event window.

# 3 Models for measuring normal performance

Two models

- Statistical models
  - Constant-mean-return model
  - Market model
  - other statistical model
- Economic models
  - CAPM

- APT

## 3.1 Statistical models

*Constant Mean-Return Model*

$$R_{it} = \mu_i + \xi_{it},$$

where $E[\xi_{it}] = 0, var[\xi_{it}] = \sigma_\xi^2$ . This is the simplest model for normal returns. Many studies show that it is difficult to improve on. For daily data, $R_{it}$ is usually measured by nominal return while with monthly data, it becomes excess return (monthly return - nominal riskfree return).
*Market Model*

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

where $E[\epsilon_{it}] = 0, var[\epsilon_{it}] = \sigma_\epsilon^2$
The market model represents a potential improvement over the constant-mean-return model. By removing the portion of the return that is related to variation in the markets return, the variance of the abnormal return is reduced.
*Other Statistical Models*
Other statistical models are factor models: i.e., market model is an example of a one factor model. The additional explanatory power in multi-factor models tends to be low.
*market-adjusted-return model*
Setting $\alpha_i = 0, \beta_i = 1$ we have $\epsilon_{it} = R_{it} - R_{mt}$ as the market-adjusted-return. This is feasible when estimation window is not available but should be used with caution.

## 3.2 Economic models

Capital Asset Pricing Models (CAPM) and the Arbitrage Pricing Theory (APT) were often used in the 70s.

## Timeline for an event study

| | estimation window | | event window | post-event window |
|---|---|---|---|---|
| $-\lvert T_0 \rvert$ | $- - - - - - - - - - - - - -$ | $\lvert T_1 \rvert$ | $- - -\lvert 0 \rvert - - -$ | $- - -\lvert T_2 \rvert - - - \lvert T_3 \rvert - - -$ |
| | | | $\tau$ | |

Notations: $\tau$ as the event date, $\tau = T_1 + 1$ to $\tau = T_2$ is the event window, and $\tau = T_0 + 1$ to $\tau = T_1$ is the estimation window. Let $L_1 = T_1 - T_0$ and $L_2 = T_2 - T_1$ be the length of the estimation window and the event window. $\tau = T_2 + 1$ to $\tau = T_3$ is the post event window and $L_3 = T_3 - T_2$ is its length. An important assumption throughout the event-study methodology is that the event is exogenous with respect to the change in market value of the security. There are examples where an event is triggered by the change in the market of security, in which case, the event is endogenous and the usual interpretation is incorrect.

# 4 Estimation of the Market Model

Let $R_t$ be an $(N \times 1)$ vector of asset returns for calendar time period $t$. $R_t$ is independently multivariate normally distributed with mean $\mu$ and covariance matrix $\Sigma$ for all $t$.
Recall that the model is

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$$

This can be expressed as a matrix regression

$$R_i = X_i \theta_i + \epsilon_i,$$

where $R_i = [R_{iT_0+1}, \cdots, R_{iT_1}]'$ is an $(L_1 \times 1))$ vector of the estimation window returns, $X_i = [i\, R_m]$ is an $(L_1 \times 2)$ matrix with a vector of ones, $i = [1, 1, \cdots, 1]'$, in the first column and the market return observations, $R_m = [R_{mT_0+1}, \cdots, R_{mT_1}]'$, in the second column, and the $\theta_i = [\alpha_i, \beta_i]'$ is the $(2 \times 1)$ parameter vector. The OLS estimators of the market-model parameters using an estimation window of $L_1$ observations are

$$\begin{aligned}
\hat{\theta}_i &= (X_i' X_i)^{-1} X_i' R_i \\
\hat{\sigma}^2_{\epsilon_i} &= \frac{1}{L_1 - 2} \hat{\epsilon}_i' \hat{\epsilon}_i \\
\hat{\epsilon}_i &= R_i - X_i \hat{\theta}_i \\
var(\hat{\theta}_i) &= (X_i' X_i)^{-1} \hat{\sigma}^2_{\epsilon_i}
\end{aligned}$$

## 4.1 Statistical Properties of Abnormal Returns

Given the market-model parameter estimates from the estimation window, it is possible to estimate and test the abnormal returns. Let $\hat{\epsilon}_i^*$ be the $(L_2 \times 1)$ sample vector of abnormal returns for firm i from the event window $T_1$ + 1 to $T_2$. Next, using the market model to measure the normal return and the OLS estimators, it is possible to generate the vector of abnormal returns

$$\begin{aligned}
\hat{\epsilon}_i^* &= R_i^* - \hat{\alpha}_i i - \hat{\beta}_i R_m^* \\
&= R_i^* - X_i^* \hat{\theta}_i
\end{aligned}$$

where $R_i^* = [R_{iT_1+1}, \cdots, R_{iT_2}]'$ is an $(L_2 \times 1)$ vector of the event-window returns, $X_i^* = [i/, R_m^*]$ is an $(L_2 \times 2)$ matrix with a vector of ones in the first column and the vector of market return observations in the second, and $\hat{\theta}_i = [\hat{\alpha}_i \hat{\beta}_i]'$ is the $(2 \times 1)$ parameter vector estimate.

Next, it is shown that the abnormal return over the event window will have conditional mean zero and covariance $V_i$:

$$\begin{aligned} E[\hat{\epsilon}_i^*|X_i^*] &= E[R_i^* - X_i^*\hat{\theta}_i|X_i^*] \\ &= E[(R_i^* - X_i^*\theta_i) - X_i^*(\hat{\theta}_i - \theta_i)|X_i^*] \\ &= 0 \end{aligned}$$

$$\begin{aligned} V_i &= E[\hat{\epsilon}_i^*\hat{\epsilon}_i^{*\prime}|X_i^*] \\ &= E[[\epsilon_i^* - X_i^*(\hat{\theta}_i - \theta_i)][\epsilon_i^* - X_i^*(\hat{\theta}_i - \theta_i)]'|X_i^*] \\ &= E[\epsilon_i^*\epsilon_i^{*\prime} - \epsilon_i^*(\hat{\theta}_i - \theta_i)'X_i^* - X_{i^*}(\hat{\theta}_i - \theta_i)\epsilon_i^{*\prime} \\ &\quad + X_i^*(\hat{\theta}_i - \theta_i)(\hat{\theta}_i - \theta_i)X_i^{*\prime}|X_i^*] \\ &= I\sigma_{\epsilon_i^*}^2 + X_i^*(X_i^*X_i)^{-1}X_i^{*\prime}\sigma_{\epsilon_i}^2 \end{aligned}$$

where $I$ is an identity matrix of size $(L_2 \times L_2)$ and the first component of the variance is due to future disturbances and the second stems from sampling error in estimating the normal return. As the length of the estimation window $L_2$ increases this drives the size of the second component down. Hence, under the null that the event has no influence on the mean or the variance of returns we can use the above equations, where $\hat{\epsilon}_i^* \sim N(0, V_i)$.

# 5 Aggregation of abnormal returns

Aggregation over time and individuals for statistical inference.
Define $CAR_i(\tau_1, \tau_2)$ as the cumulative abnormal return for security $i$ from $\tau_1$ to $\tau_2$. Let $\gamma$ be a vector of ones in positions $\tau_1 - T_1$ to $\tau_2 - T_1$ and zeros elsewhere. Then,

$$\begin{aligned} \widehat{CAR}_i(\tau_1, \tau_2) &\equiv \gamma'\hat{\epsilon}_i^* \\ Var[\widehat{CAR}_i(\tau_1, \tau_2)] &= \sigma_i^2(\tau_1, \tau_2) = \gamma' V_i \gamma \end{aligned}$$

Then, under $H_0$ ie. given event has no impact,

$$\widehat{CAR}_i(\tau_1, \tau_2) \sim N(0, \sigma_i^2(\tau_1, \tau_2))$$

The test statistics then becomes

$$\widehat{SCAR}_i(\tau_1,\tau_2) = \frac{\widehat{SCAR}_i(\tau_1,\tau_2)}{\hat{\sigma}_i(\tau_1,\tau_2)} \sim t(L_1-2)$$

Not that under $H_0, \widehat{SCAR}_i(\tau_1,\tau_2)$ has mean 0 and variance $\frac{L_1-2}{L_4-2}$. For large estimation window ($L_1 > 30$), $\widehat{SCAR}_i(\tau_1,\tau_2)$ is distributed as standard normal distribution.
Aggregation must be done over many event observations.
Assume that there is no correlation across the abnormal returns of different securities. This assumption is usually true if there is no overlap of in the event windows.
*Aggregation over securities*
Define $\bar{\epsilon}^*$ as the N-average across securities

$$\begin{aligned}
\bar{\epsilon}^* &= \frac{1}{N}\sum_{i=1}^{N}\hat{\epsilon}_i^* \\
Var(\bar{\epsilon}^*) &= V = \frac{1}{N^2}\sum_{i=1}^{N} V_i
\end{aligned}$$

*Aggregation over time*
Define $\overline{CAR}(\tau_1,\tau_2)$ as the cumulative average abnormal return from $\tau_1$ to $\tau_2$ and $\gamma$ again an $(L_2\times 1)$ vector with ones in positions $\tau_1 - T_1$ to $\tau_2 - T_1$.

$$\begin{aligned}
\overline{CAR}(\tau_1,\tau_2) &\equiv \gamma'\bar{\epsilon}^* \\
Var[\overline{CAR}(\tau_1,\tau_2)] &= \bar{\sigma}^2(\tau_1,\tau_2) = \gamma' V\gamma
\end{aligned}$$

Equivalently, to obtain $\overline{CAR}(\tau_1,\tau_2)$, we can aggregate using the sample cumulative abnormal return for each security $i$. For $N$ events, we have

$$\begin{aligned}
\overline{CAR}(\tau_1,\tau_2) &= \frac{1}{N}\sum_{i=1}^{N}\widehat{CAR}_i(\tau_1,\tau_2) \\
Var[\overline{CAR}(\tau_1,\tau_2)] &= \bar{\sigma}^2(\tau_1,\tau_2) = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2(\tau_1,\tau_2)
\end{aligned}$$

$$\overline{CAR}(\tau_1,\tau_2) \sim N(0,\bar{\sigma}^2(\tau_1,\tau_2))$$

$\bar{\sigma}^2(\tau_1,\tau_2)$ is unknown and can be consistently estimated by $\hat{\bar{\sigma}}^2(\tau_1,\tau_2) = \frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}^2(\tau_1,\tau_2)$. The, the test statistics becomes

$$J_1 = \frac{\overline{CAR}(\tau_1,\tau_2)}{[\hat{\bar{\sigma}}^2(\tau_1,\tau_2)]^{1/2}} \overset{a}{\sim} N(0,1)$$

*Second aggregation*

Define $\overline{SCAR}(\tau_1, \tau_2)$ as the average over $N$ securities from event time $\tau_1$ to $\tau_2$

$$\overline{SCAR}(\tau_1, \tau_2) = \frac{1}{N}\sum_{i=1}^{N} \widehat{SCAR}_i(\tau_1, \tau_2)$$

Assume that $N$ securities does not overlap in calendar time, then we can use the test statistics

$$J_2 = (\frac{N(L_1 - 4)}{L_1 - 2})^{1/2}\overline{SCAR}(\tau_1, \tau_2) \overset{a}{\sim} N(0, 1)$$

*Sensitivity to the normal return model*

$$\begin{aligned} \sigma^2_{\epsilon_i} &= Var[R_{it} - \alpha_i - \beta_i R_{mt}] \\ &= Var[R_{it}] - \beta_i^2 Var[R_{mt}] \\ &= (1 - R_i^2) Var[R_{it}] \end{aligned}$$

$$\begin{aligned} \sigma^2_{\xi_i} &= Var[R_{it} - \mu] = Var[R_{it}] \\ \sigma^2_{\epsilon_i} &= (1 - R_i^2)\sigma^2_{\xi_i} \end{aligned}$$

# 6 Modifying the null hypothesis

Original null hypothesis: event has no effect on mean level and variance of security returns. A modified null hypothesis would be: event has no effect on mean return only. To construct such a test statistics, we need to remove the reliance on past returns in calculating variances. We could the cross-section approach to estimating the variance:

$$\begin{aligned} \widehat{Var}[\overline{CAR}](\tau_1, \tau_2)] &= \frac{1}{N^2}\sum_{i=1}^{N}(CAR_i(\tau_1, \tau_2) - \overline{CAR}(\tau_1, \tau_2)^2 \\ \widehat{Var}[\overline{SCAR}](\tau_1, \tau_2)] &= \frac{1}{N^2}\sum_{i=1}^{N}(SCAR_i(\tau_1, \tau_2) - \overline{SCAR}(\tau_1, \tau_2)^2 \end{aligned}$$

# 7 Analysis of power

Given al alternative hypothesis $H_A$ and CDF of $J_1$, cumulative-abnormal-return-based statistic, we can tabulate the power of a test of size $\alpha$ using

$$P(\alpha, H_A) = P(J_1 < \Phi^{-1}(\alpha/2)|H_A) + P(J_1 > \Phi^{-1}(1 - \alpha/2)|H_A)$$

# 8 Nonparametric tests

$$
\begin{aligned}
J_3 &= [\frac{N^+}{N} - 0.5]\frac{N^{1/2}}{0.5} \sim N(0,1) \\
J_4 &= \frac{1}{N}\sum_{i=1}^{N}(K_{i0} - \frac{L_2+1}{2})/s(L_2) \\
s(L_2) &= \sqrt{\frac{1}{L_2}\sum_{\tau=T_1+1}^{T_2}(\frac{1}{N}\sum_{i=1}^{N}(K_{i\tau} - \frac{L_2+1}{2}))^2}
\end{aligned}
$$

where $N^+$ number of the cases where abnormal return is positive, $K_{i\tau}$ rank of the abnormal return of security $i$ for event period $\tau$.

# 9 10 steps for implementing an event study recommended by A. Williams and D. McWilliams

Step 1 Define an event that provides new information to the market

Step 2 Outline a theory that justifies a financial response to this new information

Step 3 Identify a set of firms that experience this event and identify the event dates

Step 4 Choose an appropriate event window and justify its length, if it exceeds two days.

Step 5 Eliminate or adjust for firms that experience other relevant events during the event window

Step 6 Compute abnormal returns during the event window and test their significance

Step 7 Report the percentage of negative returns and the binomial Z or Wilcoxon test statistic

Step 8 For small samples, use bootstrap methods and discuss the impact of outliers.

Step 9 Outline a theory that explains the cross-section variation in abnormal returns and test this theory econonometrically

Step 10 Report firm names and event dates in data appendix.