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General Physics I, Midterm 2
PHYS10400, Class year 99
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Solution

1.

(a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi\rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi\rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi\rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi\rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi\rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi\rho GR_e$

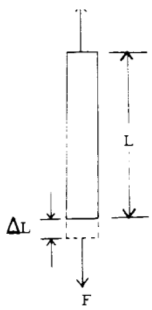
so

$$\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$$

and

$$\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times10^6\ \text{m}}{9.8\ \text{m/s}^2}} = 2.53\times10^3\ \text{s} = \boxed{42.2\ \text{min}}$$

2.



Solution: The Young's modulus of the metal bar is the ratio of longitudinal stress, F/A, to tensile strain ΔL/L (see the figure)

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

Here, A is the bar's cross-sectional area. Therefore, the elongation ΔL of the bar is

$$\Delta L = \frac{\Delta FL}{YA}$$

$$\Delta F = 2.0\ \text{tons} = 2.0\ \text{ton} \times \frac{2000\ \text{lb}}{1\ \text{ton}} = 4000\ \text{lbs.}$$

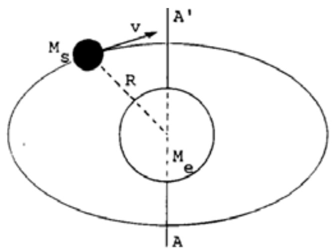
$$A = (2.0\ \text{in.} \times 1.0\ \text{in.}) = 2.0\ \text{in.}^2$$

Young's modulus for steel is $29 \times 10^6\ \text{lb/in.}^2$.

$$\Delta L = \frac{(4.0 \times 10^3\ \text{lb})(20\ \text{ft})}{(29 \times 10^6\ \text{lb/in.}^2)(2.0\ \text{in.}^2)} = 0.0014\ \text{ft}$$

$$= 0.017\ \text{in.}$$

3.



<u>Solution:</u> In the figure, the earth's gravitational pull on the satellite accounts for the satellite's centripetal acceleration:

$$\frac{M_s v^2}{R} = \frac{G M_s M_e}{R^2}$$

where M_e is the mass of the earth.

Thus: $$v = \sqrt{\frac{GM_e}{R}}$$

Its angular velocity is therefore:

$$\omega = \frac{v}{R} = \sqrt{\frac{GM_e}{R^3}}$$

Since R > > radius of satellite, when calculating the satellite's moment of inertia, with respect to the axis A-A', we can take all of its mass to be at a distance R from the axis.

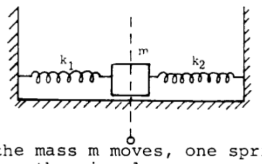
This reduces to the case of finding the moment of inertia of a point mass with respect to an axis. This is:

$$I = M_s R^2$$

The angular momentum L of the satellite is therefore:

$$L = I\omega = M_s R^2 \sqrt{\frac{GM_e}{R^3}} = M_s \sqrt{GM_e R}$$

4.



<u>Solution:</u> When the mass m moves, one spring is always stretched, and the other is always compressed by the same length. Thus:

$$\Delta x_1 = - \Delta x_2$$

where Δx_1 is the distance that the spring with force constant k_1 stretches, and Δx_2 that of the other spring. Note that a negative stretching distance represents a distance compressed. We denote the distance of the mass to the right of the origin 0 by Δx. Thus:

$$\Delta x_1 = \Delta x$$

$$\Delta x_2 = - \Delta x$$

taking positive displacement as pointing to the right. The force on the mass at any time is therefore:

$$F = - k_1 \Delta x_1 - (- k_2 \Delta x_2) = - k_1 \Delta x + k_2 (- \Delta x)$$

$$= - (k_1 + k_2) \Delta x$$

$$= - k' \Delta x$$

where we let $k' = k_1 + k_2$

Since the frequency of an oscillator having force constant k' is:

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k'}{m}}$$

then: $$\nu = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}}$$

5.

(1) During the fall $U_{initial} = K_{final}$

$$MgH = \frac{1}{2}Mv^2 \rightarrow v = \sqrt{2gH}$$

(2) During the collision, $P_i = P_f$

$$Mv = (M+m)v' \quad \text{— ②}$$

(3) Rise : $\frac{1}{2}(M+V)v'^2 = (M+m)gh$

$$v' = \sqrt{2gh}$$

$\Rightarrow$ in ②

$$M\sqrt{2gH} = (M+m)\sqrt{2gh}$$

$$M^2H = (M+m]^2h$$

$$\rightarrow h = \left[\frac{M}{M+m}\right]^2 H$$

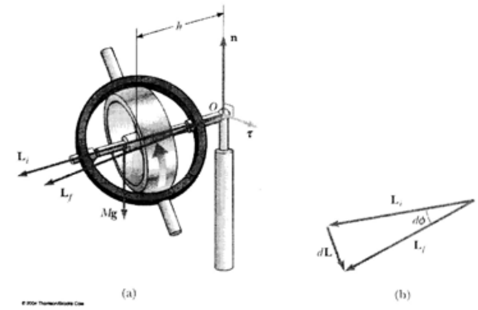
6.

7.)
   (b)......(2)
   (c) Plug in (2) in (1), (2) should setify the differential equaiton (1)
   (d)

8. (a) Kepler's law.
   (I) All planets moves in elliptical orbits, with the sun at one focus (Elliptical law)
   (II) The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals (area law)
   (III) The square of the orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit (Period law)

   (b) From the second law, $\frac{dA}{dt} = \frac{L}{2m_p} = \frac{m_p vr}{2m_p}$ =constant, where $\boldsymbol{m_p}$ is the mass of the planet, and $\boldsymbol{v}$ is the velocity at various point in the orbit. We can see ***Vr*** is a constant. So at perihelion point $\boldsymbol{V_p}$ the velocity is greater as the radius is shorter.

9. The entire situation is identical to the example as depicted in pages 326-327 in the textbook.



Mg produces a torque τ

$\tau \parallel \Delta L$

$\Delta L \parallel xz$ plane

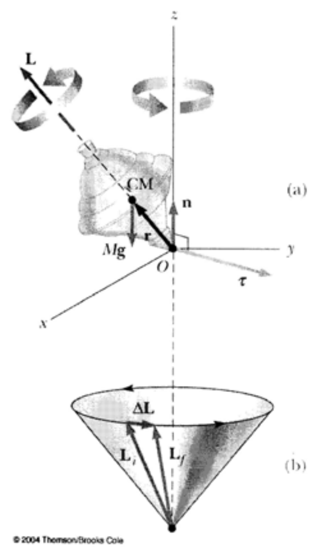
$\Delta L \perp L$ if $\Delta t \to 0$

$$\sin(d\phi) \approx d\phi = \frac{dL}{L} = \frac{\tau dt}{L} = \frac{mgh\,dt}{L}$$

$L = I\omega$

$$\therefore d\phi = \frac{mgh\,dt}{I\omega}$$

$$\boxed{\frac{d\phi}{dt} = \omega_p = \frac{mgh}{I\omega}}$$



10. In the earth field, we apply the conservation of gravitational energy to solve this question.

$\frac{1}{2}mV_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{R_{\max}} + \frac{1}{2}mV_f^2$, in our case, $\boldsymbol{V_f} = 0$ and $\boldsymbol{R_{max}} = \infty$

Therefore, $V_i = \sqrt{\frac{2GM_E}{R_E}}$. Plug in the numbers, R_E = 6370 km,

G=6.67×10^{-11} NM^2/Kg^2, ME = 5.98×10^{24} Kg

We receive, $\boldsymbol{V_i = 1.12\times10^4}$ **m/sec**