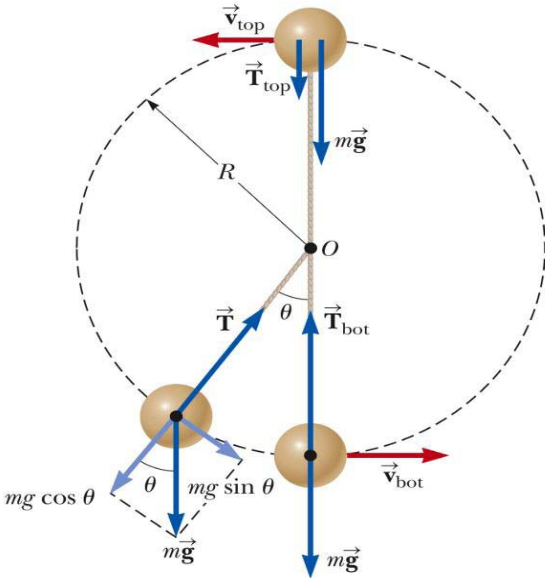
General Physics I, Midterm Exam 1 solution

1. As shown in the figure to the right, (Example 6.6 in the text book) the forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O. Forces acting on the sphere are shown when the sphere is at the top and bottom of the circle and at an arbitrary location. From the force diagram in Figure on the right, we see that the only forces acting on the sphere are the gravitational force $\mathbf{F_g} = \mathbf{mg}$ exerted by the Earth and the force $\mathbf{T}$ exerted by the cord. We resolve $\mathbf{F_g}$ into a tangential component $mg \sin\theta$ and a radial component $mg \cos\theta$.



Apply Newton's second law to the sphere in the tangential direction:

$$\sum F_g = mg\sin\theta = ma_t\,,\quad a_t = g\sin\theta$$

Apply Newton's second law to the forces acting on the sphere in the radial direction, noting that both $\mathbf{T}$ and $\mathbf{a_r}$ are directed toward O:

$$\sum F_r = T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$

Let us evaluate this result at the top and bottom of the circular path:

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right), \text{ and } T_{bottom} = mg\left(\frac{v_{bottom}^2}{Rg} + 1\right)$$

2.

Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or

$$v_i = \left(\frac{M+m}{m}\right)v_f \qquad (1)$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2$$

Thus,

$$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into (1) from above gives $\boxed{v_i = \left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

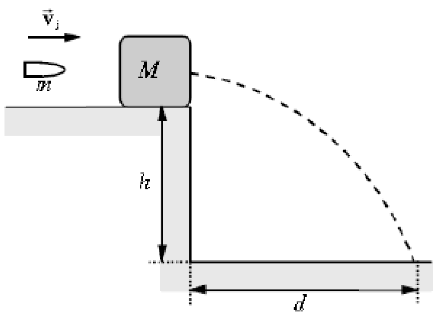


FIG. P9.53

3. (a) $F = ma \quad W = F\Delta x = ma\Delta x = m\frac{dv}{dt}dx = mvdv$. If the block moves from initially at rest to a velocity v, then the total work is, $W = \int_0^v mvdv = \frac{1}{2}mv^2$.

(b) In the Spring-Block system, the force F also does work on the block, if the block is not moving, at a displacement d, the total work will be $W = \int_0^d -kxdx = \frac{1}{2}kd^2$. This is all stored in the spring, and is call the potential of the spring-block system.

4. Using F=ma, $mg - bv = ma = m\frac{dv}{dt}$. Therefore, $\frac{dv}{dt} = g - \frac{b}{m}v$

(a) At terminal velocity, V_T, $mg - bV_T = 0$, $V_T = \frac{mg}{b}$

(b) From the above, $\frac{dv}{dt} = g - \frac{b}{m}v = g\left[1 - \frac{b}{mg}v\right]$,

$\frac{dv}{\left[1 - \frac{b}{mg}v\right]} = gt$, integrating on both sides, $-\frac{mg}{b}\ln(1 - \frac{b}{mg}v) = gt$

$\ln(1 - \frac{b}{mg}v) = -\frac{b}{mg}gt = -\frac{b}{m}t$, $\quad (1 - \frac{b}{mg}v) = e^{-\frac{b}{m}t}$

$v = \frac{mg}{b}(1 - e^{-\frac{b}{m}t})$

(c) From the answer in (b), the term b/m is called time constant. When the time constant is bigger, it reaches the terminal velocity faster. Therefore, a cat has lighter weight, and reaches terminal velocity faster, so the cat has more time to react to the fall, and may survive. However, for a human, the weight is bigger, a smaller time constant. Throughout the fall, the human may stay in acceleration stage, and will not react to the fall.

5. This problem is from Page 188 (Example 7.9) of text book.
(a) The separation of two atoms is where the potential is in its minimum. To find the minimum, we set $\frac{dU(x)}{dx} = 4\varepsilon\frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^6\right] = 4\varepsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^6}{x^7}\right] = 0$

$\frac{dU(x)}{dx} = 4\varepsilon\frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^6\right] = 4\varepsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^6}{x^7}\right] = 0$, $x = (2)^{\frac{1}{6}}\sigma$

(b) Plug in numbers given, x=2.95×10-^{10} m