

1.

In a adiabatic process, $Q=0$, for ideal gas.

$$dE_{int} = dQ - dW, \quad Q=0. \quad dW = \text{work done by the system}$$

$$nC_v dT = -p dV$$

$$n dT = -\left(\frac{p}{c_v}\right) dV, \quad \text{but } pV = nRT$$

$$\therefore p dV + V dp = nR dT$$

$$\therefore \frac{p dV + V dp}{c_p - c_v} = -\left(\frac{p}{c_v}\right) dV$$

$$n dT = \frac{p dV + V dp}{R}$$

$$= \frac{p dV + V dp}{c_p - c_v}$$

$$\frac{dp}{p} + \left(\frac{c_p}{c_v}\right) \frac{dV}{V} = 0$$

$$\ln p + \gamma \ln V = \text{constant}, \quad \gamma = \frac{c_p}{c_v}$$

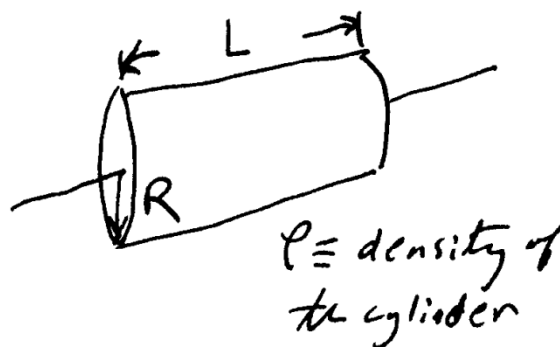
$$pV^\gamma = \text{constant}$$

$$V_2 = 0$$

2.

For a Solid Cylinder as shown on the right

$$\begin{aligned} I &= \int r^2 dm \\ &= \int r^2 2\pi r dr L \rho \\ &= 2\pi L \rho \int_0^R r^3 dr \\ &= 2\pi L \rho \frac{R^4}{4} \\ &= \frac{1}{2} M R^2 \end{aligned}$$



$$M = \pi R^2 \cdot \rho \cdot L$$

3.

2.

$$(a) \quad \frac{1}{2} m v_i^2 - \frac{GM_E m}{R_E} = - \frac{GM_E m}{r_{max}}$$

if $v_f = 0$



$$\therefore v_i^2 = 2GM_E \left(\frac{1}{R_E} - \frac{1}{r_{max}} \right)$$

→ Ans (a)

$$(b) \quad \text{if } r_{max} = \infty \quad v_i^2 = 2GM_E \left(\frac{1}{R_E} \right) = v_{esc}^2$$

$$v_{esc} = \sqrt{\frac{2MEG}{R_E}} \approx \text{plug in numbers} \approx 10 \text{ km/sec}$$

4.

Let ~~$x(t) = A \cos(\omega t + \phi)$~~ $x(t) = A \cos(\omega t + \phi)$ represents a SHO's Amplitude

$$\text{Then } v = \frac{dx(t)}{dt} = -\omega A \sin(\omega t + \phi)$$

Total energy of a SHO $E_{tot} = E_K + E_U$

$$E_K = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi)$$

$$E_U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$E_{total} = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi) + \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2} k A^2 = \text{Constant.}$$

E_{total} determined by k and Max. Amplitude A

5.

Parallel Theorem, $I = I_{cm} + MD^2$

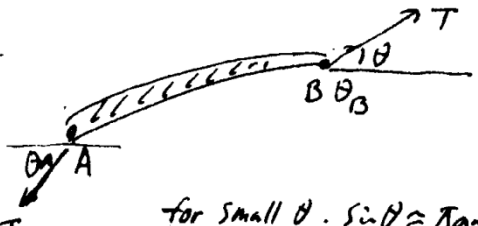
$$I = \int r^2 dm = \int (x^2 + y^2) dm \quad \text{But } \begin{aligned} x &= x' + x_{cm} \\ y &= y' + y_{cm} \end{aligned}$$

$$\begin{aligned} \therefore I &= \int [(x' + x_{cm})^2 + (y' + y_{cm})^2] dm \\ &= \int [x'^2 + 2x'x_{cm} + x_{cm}^2 + y'^2 + 2y'y_{cm} + y_{cm}^2] dm \\ &= \underbrace{\int (x'^2 + y'^2) dm}_{= I_{cm}} + \underbrace{2x_{cm} \int x' dm}_{= 0} + \underbrace{2y_{cm} \int y' dm}_{= 0} + \underbrace{(x_{cm}^2 + y_{cm}^2) \int dm}_{D^2 M} \\ &= I_{cm} + MD^2 \end{aligned}$$

definition of center of mass

6.

Use the figure on the right



The total force F_y is

$$\begin{aligned} \sum F_y &= T \sin \theta_B - T \sin \theta_A \quad \text{for small } \theta, \sin \theta \approx \tan \theta \\ &= T (\sin \theta_B - \sin \theta_A) \\ &\approx T (\tan \theta_B - \tan \theta_A) \\ &= T \left(\left. \frac{\partial y}{\partial x} \right|_{\text{at B}} - \left. \frac{\partial y}{\partial x} \right|_A \right) = m a_y = \mu \Delta x \left(\frac{\partial^2 y}{\partial t^2} \right) \end{aligned}$$

$$\therefore \mu \Delta x \left(\frac{\partial^2 y}{\partial t^2} \right) = T \left[\left(\frac{\partial y}{\partial x} \right)_B - \left(\frac{\partial y}{\partial x} \right)_A \right]$$

$$\text{or } \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\left(\frac{\partial y}{\partial x} \right)_B - \left(\frac{\partial y}{\partial x} \right)_A}{\Delta x} = \frac{\partial^2 y}{\partial x^2}$$

$$\therefore \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \quad \text{or } \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

(is the wave equation describe a wave traveling in the string.)

7.

In A Solid, in x-axis direction, due to atom vibrat

$$E_{int} = \frac{1}{2} m V_x^2 + \frac{1}{2} k x^2 \quad (\text{two degree of freedom})$$

Take into account $\hat{y}$ and $\hat{z}$ dimension. there are total of 6 degree of freedoms, then Use eqn-partition the each degree of freedom take $\frac{1}{2} k_B T$ energy

$$\therefore E_{int} = 6 \times \frac{1}{2} k_B T = 3 k_B T = 3 n R T, \text{ for } n=1$$

$$E_{int} = 3 R T \quad (\text{for } n=1). \text{ Thus } C_v = \frac{d}{dT} (E_{int}) = \underline{\underline{3R}}$$

9.

$$\text{Mean free path } \lambda = \frac{\text{total length}}{\text{number of collision}}$$

$$\lambda = \frac{\bar{v} \Delta t}{\frac{N}{V} \pi d^2 \bar{v} \Delta t} = \frac{1}{\pi d^2 \frac{N}{V}}$$

$\Rightarrow$ Assume there is no relative motions between gas molecules

For details, check Textbook

8.

(a) A free expansion process is also an adiabatic process. Since this is an irreversible process, we can find an isothermal reversible process (expansion) to calculate the entropy change of this system

$$\therefore \Delta S = \int_i^f \frac{dQ_r}{T} = \frac{1}{T} \int_i^f dQ_r. \quad dQ_r \text{ is the reversible case.}$$

in a reversible isothermal process,

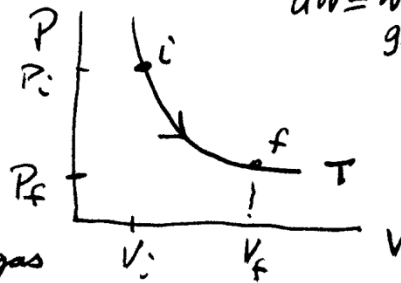
$$dE_{int} = 0 \quad \therefore dQ_r = -dW$$

$$-W = + \int_{V_i}^{V_f} P dV$$

$$= + \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

$$= nRT \ln\left(\frac{V_f}{V_i}\right)$$

= work done by the gas to the environment



$dW = \text{work done to the gas}$

$$\therefore \int_i^f dQ_r = \int_i^f -dW = -W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Thus the total entropy change is

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right)$$

(b) Since $dE_{int} = dQ_r + dW$, $dW = \text{work done on the gas}$
 $= -P dV$

$$dQ_r = dE_{int} - dW$$

$$= dE_{int} + P dV$$

$$= nC_v dT + nRT \frac{dV}{V}$$

$$\text{ii } \frac{dQ_r}{T} = nC_v \frac{dT}{T} + nR \frac{dV}{V}$$

$$\Delta S = \int_i^f \frac{dQ_r}{T} = nC_v \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$