



SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \quad \text{Time average} \quad \overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

$$x(t) = x_0 \cos(at + \phi), \text{ where } x_0 \text{ is the maximum, and } \phi \text{ is the phase angle.}$$

$$N_A = 6 \times 10^{23}, R = \text{Gas constant} = 8.31 \text{ J/mole K, room temperature} = 300\text{K, } 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa.}$$

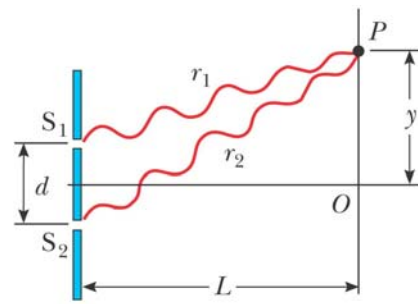
$$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}} \text{ for ideal gas.}$$

$$2\cos\phi = e^{i\phi} + e^{-i\phi}, 1 + \cos\phi = 2\cos^2(\phi/2), I \equiv E^*E, \text{ where } E \text{ is the complex electric field}$$

Problems (6 Problems, total 100%)

- Single slit diffraction:** A parallel beam of blue light (420 nm) is incident on a small aperture. After passing through the aperture, the beam is no longer parallel but diverges at 1° to the incident direction. What is the diameter of the aperture?
Note: for small angle, $\sin\theta \cong \theta$. (15%)

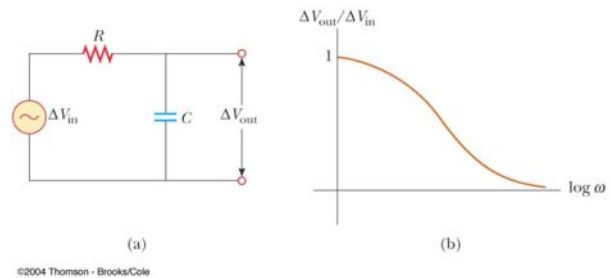
- Double-Slit interference:** Please refer to the figure on the right; (a) suppose that the electric fields from a double-slit experiment can be expressed as $\vec{E}_1 = E_0 e^{i\omega t}$ and $\vec{E}_2 = E_0 e^{i(\omega t + \phi)}$. You can treat these as the electric fields reaching the viewing screen from two slits respectively, where E_0 is the amplitude of the electric field.



You can use this to derive an expression of the intensity I on the viewing screen in terms of the phase angle ϕ . Alternatively, you can treat the interference of double-slit geometrically using trigonometry and the figure on the right. Let $E_1 = E_0 \sin(\omega t)$, and $E_2 = E_0 \sin(\omega t + \phi)$ to represent the magnitude of electric field at point P due to each wave separately. (a) Derive an expression of the intensity I on the viewing screen in terms of the phase angle ϕ . (10%) (b) What is the phase angle ϕ in terms of slit's separation, wavelength, and incident angle θ , θ is defined as the deviation of the light from the parallel line in this figure? (5%) (c) Draw the interference fringes in terms of the phase angle ϕ . (5%)

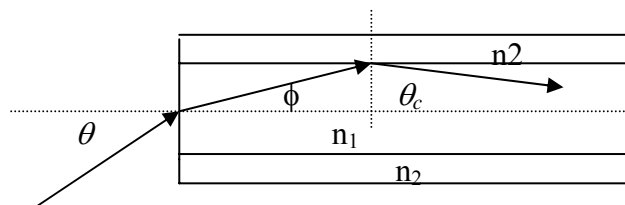
3. **Electric Filter:** Consider the filter circuit shown in the figure to the right (a) Show that the ratio of the output voltage to the input voltage is (10%)

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$



- (b) What value does this ratio approach as the frequency decreases toward zero? (5%) (c) What value does this ratio approach as the frequency increases without limit? (5%) (d) At what frequency is the ratio equal to one half? (5%)

4. **Optical Fiber and Light propagation:** An optical fiber consists of a glass core (index of refraction n_1) surrounded by a coating (index of refraction $n_2 < n_1$). Suppose a beam of light enters the fiber from air at an angle θ with the fiber axis as shown below. What is the greatest possible angle θ for which a ray can travel down the fiber? (20 %)



5. **Radiation Pressure:** Many people giving presentations use a laser pointer to direct the attention of the audience to information on the screen. If a time-averaged 3 mW pointer creates a spot on a screen that is 2.0 mm in diameter, determine the radiation pressure on a screen that reflects 100% of the light that strikes it. (10%) You can use,

$$S_{av} = \frac{P_{av}}{A}$$

6. **Basic E&M:** Write down the 4 Maxwell's Equations. (10%)