



SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \quad \text{Time average} \quad \overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$, the general solution of this equation is

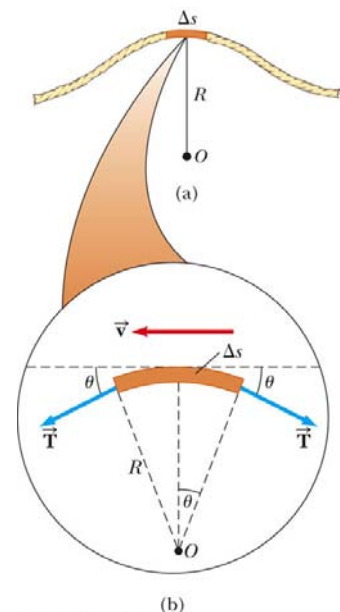
$$x(t) = x_0 \cos(at + \phi), \text{ where } x_0 \text{ is the maximum, and } \phi \text{ is the phase angle.}$$

$$N_A = 6 \times 10^{23}, R = \text{Gas constant} = 8.31 \text{ J/mole K, room temperature} = 300\text{K, } 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa.}$$

$$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}} \text{ for ideal gas.}$$

Problems (10 Problems, total 100%)

- Adiabatic process:** (10%) Prove that for an adiabatic expansion of an ideal gas, $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_v$.
- Escape velocity:** (10%) If the radius of the earth is R_E , the mass of our earth is M_E . You throw a stone of mass m with initial velocity V_i vertically up and reach a maximum height of H . (a) What is the initial velocity? (b) if you wish to throw the stone vertically up so it will reach infinite, what is the initial velocity of the stone should be?
- Pressure in a fluid:** (10%) A cylinder of cross section A and height H is vertically buried in a fluid with its top barely touch the surface. If the atmospheric pressure is P_0 , and the fluid has a density ρ . What is the pressure P measured at the bottom of the cylinder?
- Simple Harmonic Oscillator:** (10%) Take a spring of force constant k , with a mass m attached to the end. This oscillator is allowed to oscillate freely with maximum amplitude A . Prove that the total energy of a simple harmonic oscillator is constant.
- Wave speed in a string:** (10%) Use the figure on the right. A section of a string is shown in this figure. The tension in the string is T , and the line density of the string is μ . What is the speed of a wave that travels in the string?



6. **Wave equation:** (10%) In the same figure used for the previous problem. If you focused on this section of the string, you can find the mass of the string is oscillating vertically (y-direction) that is it is perpendicular to the wave's travelling direction (say, to the right or in the +x direction). Let the same section, suppose the vibration of the string can be represented as a function $y(x, t)$; a function of both x and t . Prove that the wave equation describing this wave motion is $\frac{\mu}{T} \frac{\partial^2 y(x, t)}{\partial t^2} = \frac{\partial^2 y(x, t)}{\partial x^2}$.
7. **Doppler Effect:** (10%) In a general case, the frequency shift caused by the relative motion of the sound source and sound detector is described by Doppler Effect. Let f be the original sound frequency, and f' is the detected frequency due to relative motion. The speed of the sound is V and the speed of the detector is V_D , the speed of the source is V_s . In general, the detected frequency can be expressed as $f' = f \frac{V \pm V_D}{V \mp V_s}$. If we define the relative velocity of the wave source and the detector as $u = V_s \pm V_D$. Show that this expression can also be reduced to $f' = f(1 \pm \frac{u}{V})$.
8. **Molecular specific heat:** (10%) What is the molecular specific heat C_p of an ideal gas at constant pressure?
9. **Dulong-Petit law:** (10%) Dulong-Petit law applies to general solid phase of material at higher temperature. The molecular specific law of all solids reaches $3R$ (R is the gas constant) at high temperature. The law can be explained using the equipartition energy theorem; that is in each degree of freedom, the average energy shares $1/2 k_B T$. Prove that $C_v = 3R$ in solid in high temperature.
10. **Mean Free Path:** (10%) Mean free path means the average distance an ideal gas travels before its makes collision with another gas particle. If the diameter of the gas is d , and there are total N gases particles in a large container that has volume V . What is the mean free path of these gas particles?