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General Physics I, Quiz 2
PHYS10400, Class year 98
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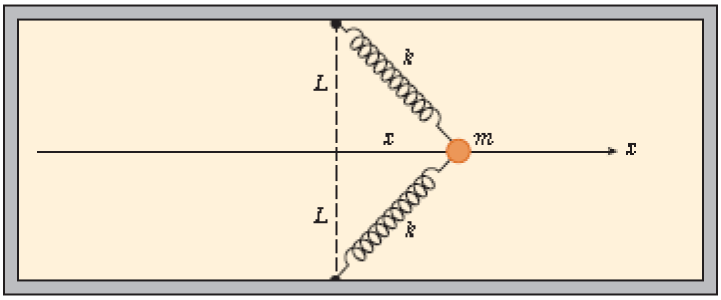
SN:_______________, Name:________________

Chapter 7-8, Serway; ABSOLUTELY NO CHEATING!
Please write the answers on the blank space or on the back of this paper to save resources.

1. A particle of mass 1.8 kg is attached between two identical springs on a horizontal, frictionless tabletop. Both springs have spring constant k and are initially unstressed. (a) The particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs as shown in Figure 1. Show that the force exerted by the springs on the particle is $\vec{F} = -2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{i}$, (b) Show that the potential energy of the system is $U(x) = kx^2 + 2kL\left(L-\sqrt{x^2+L^2}\right)$, (c) Make a plot of $U(x)$ versus x and identify all equilibrium points. Assume L=1.20 m and k = 40.0 N/m, (d) If the particle is pulled 0.500 m to the right and then released, what is its speed when it reaches the equilibrium point x = 0?



Top View

(a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

$$\vec{\mathbf{F}} = -2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}$$

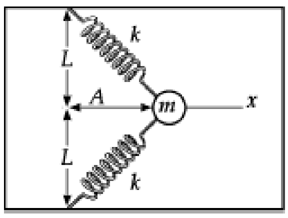


FIG. P7.43

(b) Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2 + 1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

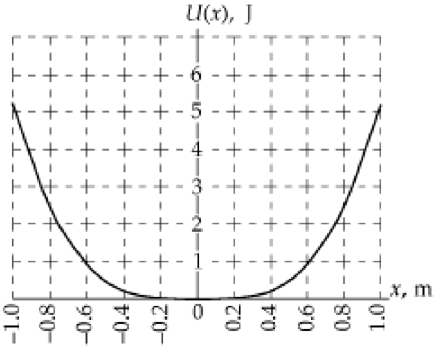


FIG. P7.43(c)

(d) $K_i + U_i + \Delta E_{mech} = K_f + U_f$

$0 + 0.400\ \text{J} + 0 = \frac{1}{2}(1.8\ \text{kg})v_f^2 + 0$

$v_f = \boxed{0.666\ \text{m/s}}$

2. A 2.00-kg particle moves along the x axis. Its position varies with time according to $x = t + 2.0t^3$, where x is in meters and t is in seconds. Find (a) the kinetic energy at any time t, (b) the acceleration of the particle and the force acting on it at time t, (c) the power being delivered to the particle at time t, and (d) the work done on the particle in the interval t = 0 to t = 2.00 s.

(a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(1.00 + 12.0t^2 + 36.0t^4\right)\ \text{J}}$$

(b) $a = \frac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 2.00(12.0t) = \boxed{(24.0t)\ \text{N}}$

(c) $\mathcal{P} = Fv = (24.0t)\left(1 + 6.00t^2\right) = \boxed{\left(24.0t + 144t^3\right)\ \text{W}}$

(d) $W = \int_0^{2.00} \mathcal{P}\,dt = \int_0^{2.00} \left(24.0t + 144t^3\right)dt = \boxed{624\ \text{J}}$