1. $\vec{E}_{total}^2 = E \cdot E = (\vec{E}_1 + \vec{E}_2)\cdot(\vec{E}_1 + \vec{E}_2) = E_1^2 + E_2^2 + 2\vec{E}_1 \cdot \vec{E}_2$, $E_1^2 = I_1$, $E_2^2 = I_2$

(1) $E_1 \parallel E_2$. $\vec{E}_{tot}^2 = I_1 + I_2 + I_{12}$ $\quad I_{12} = 2\langle E_1 \cdot E_2 \rangle_T$

(2) $E_1 \perp E_2$, $E_{tot}^2 = I_1 + I_2$

$$\langle \vec{E}_1 \cdot \vec{E}_2 \rangle_T = \langle \vec{E}_{01} \cdot \vec{E}_{02} [\cos(\vec{k}_1 \cdot \vec{r} - \omega t + \varepsilon_1) \times \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \varepsilon_2)) \rangle_T$$

$$= \frac{1}{2} \vec{E}_{01} \vec{E}_{02} \cos(\underbrace{\vec{k}_1 \cdot \vec{r} + \varepsilon_1 - \vec{k}_2 \cdot \vec{r} - \varepsilon_2}_{\delta})$$

$$= \frac{1}{2} \vec{E}_{01} \cdot \vec{E}_{02} \cos\delta$$ (depends on the angle between $\vec{E}_{01}$ and $\vec{E}_{02}$)

① If $E_1 \perp E_2$. $I_{12} = 0$.

② If $E_1 \parallel E_2$ $I_{12} = E_{01}E_{02}\cos\delta$, $I_1 = \langle E_1^2 \rangle = \frac{1}{2}E_{01}^2$, $I_2 = \langle E_2^2 \rangle_T = \frac{1}{2}E_{02}^2$

$I_{12} = 2\sqrt{I_1 I_2}\,\delta 2$ $\quad \therefore \; I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

2.

(1) $\delta = \delta(x - x_0)$ $\quad x = x_0$. $\delta(x_0) = 1$ otherwise $\delta(x) = 0$

$$\int_{-\infty}^{\infty} \delta(x - x_0) e^{-ikx} dx = e^{ikx_0}$$

(2) Let $f(x) = \begin{cases} 1 & |x| < b \\ 0 & |x| > 0 . \end{cases}$

$$\int_{-\infty}^{\infty} f(x') e^{-ikx'} dx' = \frac{1}{ik}\left[e^{-ikx}\right]_{-b}^{b} = \frac{1}{ik}\,\frac{e^{ikb} - e^{-ikb}}{1}$$

$$= \frac{2}{k}\,\frac{e^{ikb} - e^{-ikb}}{2i} = \frac{2}{k}\sin kb = 2b\,\frac{\sin kb}{kb}$$

$$= 2b\,\mathrm{sinc}(kb)$$

3

A monochromatic light of frequency ν_1, intensity I_0

$$E_1(\nu_1) = \frac{E_0}{\sqrt{4}} e^{2\pi i \nu_1 d}$$

$$E_2(\nu_1) = \frac{E_0}{\sqrt{4}} e^{2\pi i \nu_1 (d+x)} .$$

$$E_{tot} = E_1 + E_2$$

The intensity of the radiation can be taken as

$$(1) \quad I(\nu) \propto E_{tot}^*(\nu) \cdot E_{tot}(\nu) .$$

The intensity detected is

$$I_{tot}(\delta) = E_{tot}^*(\delta) \cdot E_{tot}(\delta)$$

$$= \frac{1}{4} E_0^2 [1 + e^{-2\pi i \nu_1 x}] \cdot [1 + e^{2\pi i \nu_1 x}]$$

$$= \frac{I_0}{2} + \frac{I_0}{2} \cos(2\pi\nu_1 x) .$$

3. $E_1(\nu_1) = \frac{E_0}{\sqrt{4}} e^{2\pi i \nu_1 d}$; $E_2(\nu_1) = \frac{E_0}{\sqrt{4}} e^{2\pi i \nu_1 (d+x)}$

(1)

$$E_{tot} = E_1 + E_2$$

$$= \frac{E_0}{\sqrt{4}} \left[e^{2\pi i \nu_1 d} + e^{2\pi i \nu_1 d} e^{2\pi i \nu_1 x} \right]$$

$$= \frac{E_0}{\sqrt{4}} e^{2\pi i \nu_1 d} \left[1 + e^{2\pi i \nu_1 x} \right]$$

$$I_{tot}(x) = E_{tot}^*(x) \, E_{tot}(x)$$

$$= \frac{E_0}{\sqrt{4}} \cancel{e^{-2\pi i \nu_1 d}} \left[1 + e^{-2\pi i \nu_1 x} \right] \frac{E_0}{\sqrt{4}} \cancel{e^{2\pi i \nu_1 d}} \left[1 + e^{2\pi i \nu_1 x} \right]$$

$$= \frac{1}{4} E_0^2 \left[1 + e^{-2\pi i \nu_1 x} + e^{2\pi i \nu_1 x} + 1 \right] \qquad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$= \frac{1}{4} E_0^2 \left[2 + 2 \cdot \frac{e^{2\pi i \nu_1 x} + e^{-2\pi i \nu_1 x}}{2} \right]$$

$$= \frac{1}{2} E_0^2 \left[1 + \cos(2\pi \nu_1 x) \right]$$

$$= \frac{1}{2} I_0 \left[1 + \cos(2\pi \nu_1 x) \right]$$

Define $I_{tot}(x) - \frac{1}{2} I_0 = I_{detected}$

$$= \frac{1}{2} I_0 + \frac{1}{2} I_0 \cos(2\pi \nu_1 x) \Rightarrow$$

(2).

$$\int_{-L}^{+L} I_{det}(x) \, dx$$

$$= \underbrace{\cancel{\int_{-L}^{+L} \frac{1}{2} I_0 \, dx}}_{①} + \underbrace{\int_{-L}^{+L} \frac{1}{2} I_0 \cos(2\pi \nu_1 x) \, dx}_{②}$$

3 (2). ~~for ① $\int_{-L}^{+L} \frac{1}{2} I_0 \, dx = I_0 L$~~ X

for ② $\int_{-L}^{+L} \frac{1}{2} I_0 \cos(2\pi\nu_1 x)\cos(2\pi\nu x)\, dx$

$$= \frac{1}{2} I_0 \int_{-L}^{+L} \frac{1}{2}\left[\cos 2\pi(\nu_1+\nu)x + \cos 2\pi(\nu_1-\nu)x\right] dx$$

$$= \frac{1}{4} I_0 \left[\frac{\sin 2\pi(\nu_1+\nu)x}{2\pi(\nu_1+\nu)} + \frac{\sin 2\pi(\nu_1-\nu)x}{2\pi(\nu_1-\nu)}\right]_{-L}^{L}$$

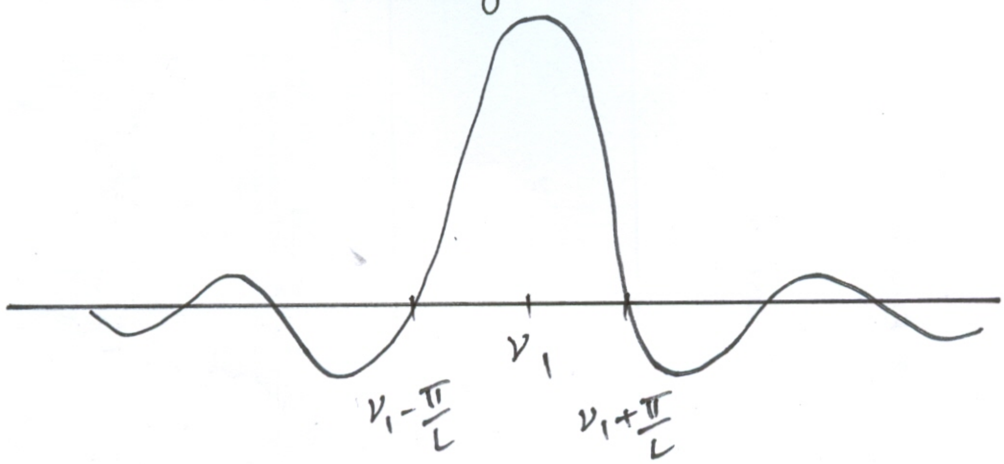
$$= \frac{1}{2} I_0 \left[\frac{\sin 2\pi(\nu_1+\nu)L}{2\pi(\nu_1+\nu)} + \frac{\sin 2\pi(\nu_1-\nu)L}{2\pi(\nu_1-\nu)}\right]$$

$$= \frac{1}{2} I_0 L\left[\operatorname{sinc} 2\pi(\nu_1+\nu) + \operatorname{sinc} 2\pi(\nu_1-\nu)\right] \quad \operatorname{sinc} x = \frac{\sin x}{x}$$

$$= \frac{1}{2} I_0 L\left[\operatorname{sinc}(\nu_1+\nu) + \operatorname{sinc}(\nu_1-\nu)\right]$$

(3). Since $\operatorname{sinc}(\nu_1+\nu) \approx 0$ $\therefore$ $\mathcal{F}_i\left(I_{det}(x)\right) = \frac{1}{2} I_0 L \operatorname{sinc}(\nu_1-\nu)$

plotted as the following!



4. $dE=\frac{\varepsilon_L}{R}\sin(\omega t-kr)\,dy$; $r=R-y\sin\theta+\frac{y^2}{2R}\cos^2\theta+\cdots$

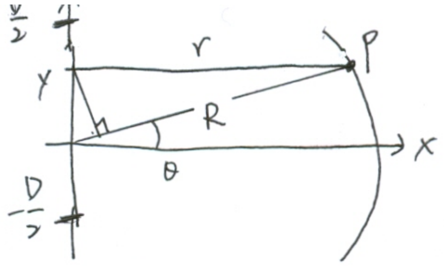
可略.

$\Rightarrow dE=\frac{\varepsilon_L}{R}\sin[\omega t-k(R-y\sin\theta)]\,dy$

$\Rightarrow E=\frac{\varepsilon_L}{R}\int_{-\frac{D}{2}}^{+\frac{D}{2}}\sin(\omega t-kR+ky\sin\theta)\,dy$

$=\frac{\varepsilon_L}{R}\times\frac{1}{k\sin\theta}\Big[-\cos(\omega t-kR+ky\sin\theta)\Big]_{-\frac{D}{2}}^{+\frac{D}{2}}$

$=\frac{\varepsilon_L}{R}\times\frac{1}{k\sin\theta}\Big[-\cos(\omega t-kR+k\frac{D}{2}\sin\theta)$
$+\cos(\omega t-kR-k\frac{D}{2}\sin\theta)\Big]$



$r^2=R^2+y^2-2Ry\cos(90^\circ-\theta)$

$r^2=R^2+y^2-2Ry\sin\theta$

$\frac{r^2}{R^2}=1-2\frac{y}{R}\sin\theta+(\frac{y}{R})^2$

$\frac{r}{R}=\Big[1-2\frac{y}{R}\sin\theta+(\frac{y}{R})^2\Big]^{\frac{1}{2}}$

$=1-\frac{y}{R}\sin\theta+\frac{1}{2}\frac{y^2}{R^2}\cos^2\theta+\cdots$

$r=R-y\sin\theta+\frac{y^2}{2R}\cos^2\theta+\cdots$

利用和角公式展開：

$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

$=\frac{\varepsilon_L}{R}\times\frac{1}{k\sin\theta}\Big[-\cos(\omega t-kR)\cos(k\frac{D}{2}\sin\theta)+\sin(\omega t-kR)\sin(k\frac{D}{2}\sin\theta)$
$+\cos(\omega t-kR)\cos(k\frac{D}{2}\sin\theta)+\sin(\omega t-kR)\sin(k\frac{D}{2}\sin\theta)\Big]$

$=\frac{\varepsilon_L}{R}\times\frac{1}{k\sin\theta}\times 2\times\sin(\omega t-kR)\sin(k\frac{D}{2}\sin\theta)$

$\boxed{\begin{aligned}&\sin(-\theta)\\&=-\sin\theta\\&\cos(-\theta)\\&=\cos\theta\end{aligned}}$

$=\frac{\varepsilon_L}{R}D\,\frac{\sin(\omega t-kR)\sin(k\frac{D}{2}\sin\theta)}{k\frac{D}{2}\sin\theta}$ ，令 $k\frac{D}{2}\sin\theta=\beta$

$=\frac{\varepsilon_L}{R}D\,\frac{\sin\beta}{\beta}\sin(\omega t-kR)$

$I(0)=\frac{1}{2}(\frac{\varepsilon_L}{R}D)^2$

$I(\theta)=\langle E^2\rangle_T$

$=\langle(\frac{\varepsilon_L}{R}D)^2(\frac{\sin\beta}{\beta})^2\sin^2(\omega t-kR)\rangle$

$=(\frac{\varepsilon_L}{R}D)^2(\frac{\sin\beta}{\beta})^2\langle\sin^2(\omega t-kR)\rangle$

$=\frac{1}{2}(\frac{\varepsilon_L}{R}D)^2(\frac{\sin\beta}{\beta})^2=I(0)(\frac{\sin\beta}{\beta})^2$

5、 $$E = \underbrace{c\int_{-\frac{b}{2}}^{\frac{b}{2}} F(z)\,dz}_{①} + \underbrace{c\int_{a-\frac{b}{2}}^{a+\frac{b}{2}} F(z)\,dz}_{②} \quad ; \; F(z) = \sin(\omega t - kR + kz\sin\theta)$$

$$① = c\int_{-\frac{b}{2}}^{\frac{b}{2}} \sin(\omega t - kR + kz\sin\theta)\,dz = \frac{c}{k\sin\theta}\Big[-\cos(\omega t - kR + kz\sin\theta)\Big]_{z=-\frac{b}{2}}^{\frac{b}{2}}$$

$$= \frac{c}{k\sin\theta}\left[\begin{array}{l} -\cos(\omega t-kR)\cos(k\frac{b}{2}\sin\theta) + \sin(\omega t-kR)\sin(k\frac{b}{2}\sin\theta) \\ +\cos(\omega t-kR)\cos(k\frac{b}{2}\sin\theta) + \sin(\omega t-kR)\sin(k\frac{b}{2}\sin\theta) \end{array}\right]$$

$$= \frac{2c}{k\sin\theta}\sin(\omega t-kR)\sin(k\frac{b}{2}\sin\theta)$$

$$= cb\,\frac{\sin(k\frac{b}{2}\sin\theta)}{\frac{kb}{2}\sin\theta}\sin(\omega t-kR) = cb\,\frac{\sin\beta}{\beta}\sin(\omega t-kR)$$

$$② = c\int_{a-\frac{b}{2}}^{a+\frac{b}{2}} F(z)\,dz = \frac{c}{k\sin\theta}\Big[-\cos(\omega t-kR+kz\sin\theta)\Big]_{z=a-\frac{b}{2}}^{a+\frac{b}{2}}$$

$$= \frac{c}{k\sin\theta}\Big[-\cos(\omega t-kR+ak\sin\theta+\frac{b}{2}k\sin\theta) + \cos(\omega t-kR+ak\sin\theta-\frac{b}{2}k\sin\theta)\Big]$$

$$= \frac{c}{k\sin\theta}\left[\begin{array}{l} -\cos(\omega t-kR)\cos(ak\sin\theta+\frac{b}{2}k\sin\theta) + \sin(\omega t-kR)\sin(ak\sin\theta+\frac{b}{2}k\sin\theta) \\ +\cos(\omega t-kR)\cos(ak\sin\theta+\frac{b}{2}k\sin\theta) - \sin(\omega t-kR)\sin(ak\sin\theta+\frac{b}{2}\sin\theta) \end{array}\right]$$

$$= \frac{c}{k\sin\theta}\left\{\begin{array}{l} -\cos(\omega t-kR)\big[\cos(ak\sin\theta)\cos(\frac{b}{2}k\sin\theta) - \sin(ak\sin\theta)\sin(\frac{b}{2}k\sin\theta)\big] \\ +\cos(\omega t-kR)\big[\cos(ak\sin\theta)\cos(\frac{b}{2}k\sin\theta) + \sin(ak\sin\theta)\sin(\frac{b}{2}k\sin\theta)\big] \\ +\sin(\omega t-kR)\big[\sin(ak\sin\theta)\cos(\frac{b}{2}k\sin\theta) + \cos(ak\sin\theta)\sin(\frac{b}{2}k\sin\theta)\big] \\ -\sin(\omega t-kR)\big[\sin(ak\sin\theta)\cos(\frac{b}{2}k\sin\theta) - \cos(ak\sin\theta)\sin(\frac{b}{2}k\sin\theta)\big] \end{array}\right\}$$

$$= \frac{2c}{k\sin\theta}\times\sin(\frac{b}{2}k\sin\theta)\Big[\cos(\omega t-kR)\sin(ak\sin\theta) + \sin(\omega t-kR)\cos(ak\sin\theta)\Big]$$

$$= cb\,\frac{\sin(\frac{kb}{2}\sin\theta)}{\frac{kb}{2}\sin\theta}\sin(\omega t-kR+2\times\frac{ka}{2}\sin\theta)$$

令 $\alpha = \frac{ka}{2}\sin\theta$.

$$= cb\,\frac{\sin\beta}{\beta}\sin(\omega t-kR+2\alpha)$$

得 $E = ①+② = bc\,\frac{\sin\beta}{\beta}\Big[\sin(\omega t-kR) + \sin(\omega t-kR+2\alpha)\Big]$

5、

$$E = bc\frac{\sin\beta}{\beta}\left[\sin(wt-KR)+\underbrace{\sin(wt-KR+2\alpha)}\right]$$

$$\begin{aligned}
&\sin(wt-KR+2\alpha)\\
&=\sin(wt-KR)\cos(2\alpha)+\cos(wt-KR)\sin(2\alpha)\\
&=\sin(wt-KR)(2\cos^2\alpha-1)+\cos(wt-KR)(2\sin\alpha\cos\alpha)\\
&=2\cos\alpha\left[\sin(wt-KR)\cos\alpha-\frac{\sin(wt-KR)}{2\cos\alpha}+\cos(wt-KR)\sin\alpha\right]\\
&=2\cos\alpha\left[\sin(wt-KR+\alpha)-\frac{\sin(wt-KR)}{2\cos\alpha}\right]\\
&=2\cos\alpha\sin(wt-KR+\alpha)-\sin(wt-KR)
\end{aligned}$$

$$\therefore E = bc\frac{\sin\beta}{\beta}\left[\sin(wt-KR)+2\cos\alpha\sin(wt-KR+\alpha)-\sin(wt-KR)\right]$$

$$= 2bC\frac{\sin\beta}{\beta}\cos\alpha\sin(wt-KR+\alpha)$$

其中 $I(0)=\frac{1}{2}\left(\frac{\varepsilon_L}{R}D\right)^2$ …由第四題算得，其中 $\frac{\varepsilon_L}{R}\equiv C$

$$\begin{aligned}
I(\theta)&=\langle E^2\rangle_T\\
&=4b^2C^2\frac{\sin^2\beta}{\beta^2}\cos^2\alpha\underbrace{\langle\sin^2(wt-KR+\alpha)\rangle_T}_{=\frac{1}{2}}\\
&=2b^2C^2\frac{\sin^2\beta}{\beta^2}\cos^2\alpha\\
&=4I_0\left(\frac{\sin^2\beta}{\beta^2}\right)\cos^2\alpha
\end{aligned}$$