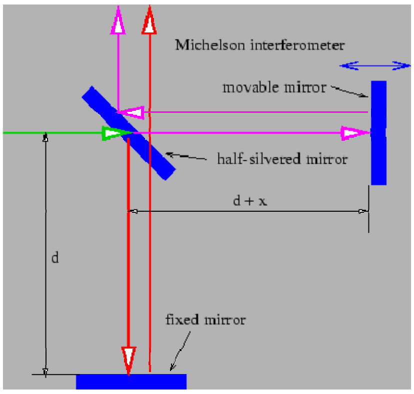
Department of Physics
PHYS30100, Class year 97

Modern Optics, Midterm Exam

1. (Total 10 points) In a two-beam interference situation, suppose the electric field of the i^th beam is $\vec{E}_i(r,t) = E_{0i}\cos(\vec{k}_i \cdot \vec{r} - \omega t + \varepsilon_i)$, where i =1, 2 in our case. Derive the interference intensity for cases, (1) E_1 is parallel to E_2, (2) E_1 is perpendicular to E_2. Define any needed parameters if it is not given in the problem. (10%)
2. (Total 10 points)What is the Fourier transform of (1) a delta function, (2) a square function? You can assume the parameters you need. Note: a typical square function have value of unity in a certain range, say a≤ ***x*** ≤ b.
3. (Total 40 points) **<u>Interference</u>** This is a problem to test you if you understand the concept of interference. When two light beams are traveling at slight different optical paths, when they combine, the electric field can be added together. The **Michelson interferometer** is the most common configuration for optical interferometry that and was invented by Albert Abraham Michelson. An interference pattern is produced by splitting a beam of light into two paths, bouncing the beams back and recombining them. The different paths may be of different lengths or be composed of different materials to create alternating interference fringes on a back detector. Michelson, along with Edward Morley, used this interferometer for the famous Michelson-Morley experiment (1887) in which this interferometer was used to prove the non-existence of the luminiferous aether. A schematic drawing is depicted in the right figure. As shown in this figure, one electric beam coming from the left, hits the beam splitter and splitted into two beams. One of them goes through a fixed mirror (the bottom mirror) travels at a distance ***d*** and the other goes through a moving mirror (the right mirror) travels at a distance ***d+x***. When they recombine in the detector (shown on the top of this figure), the two beam undergone difference paths will combine and produce an interference pattern (or the recombined electric field). Suppose the two beams are represented by



$$E_1(\nu_1) = \frac{E_0}{\sqrt{4}} e^{i2\pi\nu_1 d} \text{ and } E_1(\nu_1) = \frac{E_0}{\sqrt{4}} e^{i2\pi\nu_1 (d+x)}$$

, respectively. Here, ν_1 is the monochromatic light frequency incoming from the left of the interferometer. (1) What is the detected intensity of the recombined lights at the detector as a function of the moving mirror's traveling distance, $I_{det}(x)$? (20%) The detected intensity is a function of the moving mirror's traveling distance x. (2) You will see, if you Fourier transform the detected intensity $I_{det}(x)$, you will recover the frequency of the incoming light. Do the Fourier transform and suppose the mirror travels a total distance of ***L*** (from –***L*** to +***L***) (10%). (3) Draw the resultant Fourier transform from (2), and why it is not a delta function (10%)? This is the theory of a Fourier transform Infrared Spectrometer (FTIR), if you use infrared as you light source.

4. (Total 20 points) **Fraunhofer diffraction (single slit)** Use the following figure, for a coherence light source, if the point of observation is very far from the coherent line source and R>>D. The field due to the differential segment of the source ***dy*** is $dE = \frac{\varepsilon_L}{R}\sin(\omega t - kr)dy$. Show, carefully, the observed diffraction pattern in the observation plane to be $I(\theta) = \frac{1}{2}(\frac{\varepsilon_L D}{R})^2(\frac{\sin\beta}{\beta})^2$, where $\beta \equiv (\frac{kD}{2})\sin\theta$. (20%)

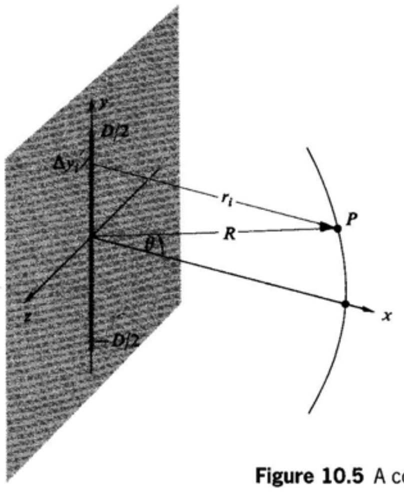


Figure 10.5 A coherent line source.

5. (Total 20 points) **Fraunhofer diffraction (double slit)** Derive in the same conditions, but for a double slits situation the diffraction pattern is $I(\theta) = 4I_0(\frac{\sin\beta}{\beta})^2\cos^2\alpha$. (20%)