4.46 $\sin x = x - x^3/3! + x^5/5! - \ldots$ and so $\sin(\alpha \pm \beta) = (\alpha \pm \beta)[1 - (\alpha \pm \beta)^2/6]$ using Snell's Law $\theta_t(1 - \theta_t^2/6 + \ldots) = (\theta_i/n)(1 - \theta_i^2/6 + \ldots)$. Use $1\theta_i = n\theta_t$ and the fact that when x is very small $(1+x)^{-1} \approx 1 - x$ we have $\theta_t = (\theta_i/n)(1 - \theta_i^2/6)(1 + \theta_i^2/6n^2)$ dropping terms higher than the third power in θ_i we get $\theta_t = (\theta_i/n)[1 - (n^2-1)\theta_i^2/6n^2]$ and so

$$\theta_i \pm \theta_t = \theta_i \left[1 \pm \frac{1}{n}\left(1 - \frac{n^2-1}{6n^2}\theta_i^2\right)\right].$$

Using Eq. (4.42) and the power series representation of the sine, where terms higher than the third power in θ_i are dropped,

$$-r_\perp = \frac{n - 1 + \frac{\theta_i^2}{6n^2}[n^2 - 1 - (n-1)^3]}{n + 1 - \frac{\theta_i^2}{6n^2}[n^2 - 1 + (n-1)^3]} = \left(\frac{n-1}{n+1}\right)\left(1 + \frac{\theta_i^2}{n}\right)$$

4.49 Compute $dr_\perp/d\theta_i$ at $\theta_i = 90°$; we'll use $d\theta_t/d\theta_i = 0$ and then prove it; taking the derivative of Eq. (4.42) we get

$$\begin{aligned} dr_\perp/d\theta_i &= -\cos(\theta_i - \theta_t)/\sin(\theta_i + \theta_t) \\ &\quad + \sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t)/\sin^2(\theta_i + \theta_t) \end{aligned}$$

and for $\theta_i = 90°$ this becomes

$$dr_\perp/d\theta_i = -\sin\theta_t/\cos\theta_t - \sin\theta_t\cos\theta_t/\cos^2\theta_t = 2\tan\theta_t$$

and using Snell's Law, i.e, $\sin\theta_t = 1/n$ when $\theta_i \approx 90°$, and the fact that

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t},$$

$$dr_\perp/d\theta_i = 2\tan\theta_t = 2\sin\theta_t/\cos\theta_t = 2/n\cos\theta_t = 2/\sqrt{n^2-1}:$$

this is the rise over the run at the end of the curve where $\theta_i \approx 90°$. Thus if $\alpha_\perp$ is the angle made ith the vertical $\tan\alpha_\perp = \sqrt{n^2-1}/2$.

4.77

$$t_\| = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

$$t'_\| = \frac{2\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_2-\theta_1)}$$

$$t_\| t'_\| = \frac{\sin 2\theta_1 \cos 2\theta_2}{\sin^2(\theta_1+\theta_2)\cos^2(\theta_1-\theta_2)} = T_\|$$

from Eq. (4.100). Similarly $t_\perp t'_\perp = T_\perp$.

$$r_\|^2 = \left[\frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}\right]^2 = \left[\frac{-\tan(\theta_2-\theta_1)}{\tan(\theta_1+\theta_2)}\right]^2$$

$$r_\|'^2 = \left[\frac{\tan(\theta_2-\theta_1)}{\tan(\theta_1+\theta_2)}\right]^2 = r_\|^2 = R_\|.$$

8.17 I' = flux density through middle polarizer $= I_1 \cos^1 \theta$ (8.24).

$$I' = I_1 \cos^2(45°) = I_1/2$$
$$I_2 = I' \cos^2(45°)$$
$$= (I_1/2)/2 = I_1/4$$

8.52 $J = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos 45° \\ \sin 45° \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

8.58 $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Therefore light polarized at 45° is unchanged, as expected.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

A horizontal $\mathcal{P}$-state is changed to an $\mathcal{R}$ state.