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Department of Physics
PHYS30100, Class year 97

Modern Optics, Quiz 1

SN:________________, Name: Key

1. In a two-beam interference situation, suppose the electric field of the i^{th} beam is $\vec{E_i}(r,t) = E_{0i}\cos(\vec{k_i}\cdot\vec{r} - \omega t + \varepsilon_i)$, where i =1, 2 in our case. Derive the interference intensity for cases, (1) E_1 is parallel to E_2, (2) E_1 is perpendicular to E_2. Define any needed parameters if it is not given in the problem.

$$\vec{E}^2_{total} = E\cdot E = (\vec{E_1}+\vec{E_2})\cdot(\vec{E_1}+\vec{E_2}) = E_1^2 + E_2^2 + 2\vec{E_1}\cdot\vec{E_2}, \quad E_1^2 = I_1,\ E_2^2 = I_2$$

(1) $E_1 \parallel E_2$, $\vec{E}^2_{tot} = I_1 + I_2 + I_{12}$ $\qquad I_{12} = 2\langle E_1\cdot E_2\rangle_T$

(2) $E_1 \perp E_2$, $E_{tot}^2 = I_1 + I_2$

$$\langle\vec{E_1}\cdot\vec{E_2}\rangle_T = \langle\vec{E_{01}}\cdot\vec{E_{02}}[\cos(\vec{k_1}\cdot\vec{r} - \omega t + \varepsilon_1)\times\cos(\vec{k_2}\cdot\vec{r} - \omega t + \varepsilon_2)\rangle_T$$

$$= \frac{1}{2}\vec{E_{01}}\vec{E_{02}}\cos(\underbrace{\vec{k_1}\cdot\vec{r} + \varepsilon_1 - \vec{k_2}\cdot\vec{r} - \varepsilon_2}_{\delta})$$

$$= \frac{1}{2}\vec{E_{01}}\cdot\vec{E_{02}}\cos\delta \quad \text{(depends on the angle between } \vec{E_{01}} \text{ and } \vec{E_{02}})$$

① If $E_1 \perp E_2$, $I_{12} = 0$.

② If $E_1 \parallel E_2$ $I_{12} = E_{01}E_{02}\cos\delta$, $I_1 = \langle E_1^2\rangle = \frac{1}{2}E_{01}^2$, $I_2 = \langle E_2^2\rangle_T = \frac{1}{2}E_{02}^2$

$I_{12} = 2\sqrt{I_1 I_2}\,\delta 2$ $\quad\therefore\ I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

2. What is the Fourier transform of (1) a delta function, (2) a square function? You can assume the parameters you need. Note: a typical square function have value of unity in a certain range, say $a \le x \le b$.

(1) $\delta = \delta(x-x_0)$ $\quad x = x_0.\ \delta(x_0) = 1$ otherwise $\delta(x) = 0$

$$\int_{-\infty}^{\infty}\delta(x-x_0)e^{-ikx}dx = e^{ikx_0}$$

(2) Let $f(x) = \begin{cases} 1 & |x| < b \\ 0 & |x| > 0. \end{cases}$

$$\int_{-\infty}^{\infty} f(x')e^{-ikx'}dx' = \frac{1}{ik}\left[e^{-ikx}\right]_{-b}^{b} = \frac{1}{ik}\,\frac{e^{ikb} - e^{-ikb}}{1}$$

$$= \frac{2}{k}\,\frac{e^{ikb} - e^{-ikb}}{2i} = \frac{2}{k}\sin kb = 2b\frac{\sin kb}{kb}$$

$$= 2b\,\mathrm{sinc}(kb)$$