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General Physics I, Final 1
PHYS1000AA, AB, AC, Class year 114-1
12-23-2025; 12-30-2025

SN:____________________, Name:____________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are NOT allowed.

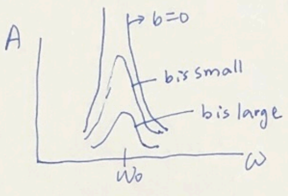
Problems (6 Problems, total 100%) The final examines will take place with two one-hr examines each is 50% of the final grades.

1. **Forced oscillation: (20%)** In a forced oscillation, if you applied a force $F_0 Sin\omega t$ to the oscillation, where the displacement of the oscillation can be represented as $x(t) = Acos\,(\omega t + \phi)$, after applied the force, the amplitude A can be calculated as $A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2+(\frac{b\omega}{m})^2}}$. (a) Draw the amplitude as function of the applied frequency ω is near the natural frequency and when the damping is very large, small and b=0. (b) Briefly explain your answer in (a).

1、 (a) $X = A\cos(\omega t+\phi)$

$$A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2+(\frac{b\omega}{m})^2}}$$

When the applied oscillation is near ω_0.
depending on the value of the damping ~~Fea~~ term b.
there are 3 posibilities. b=small, b is large. b=0



(b) When ω is near the natural frequency ω_0
the Amplitude depends on the damping term
$(\frac{b\omega}{m})^2$, $A = \frac{F_0/m}{(\frac{b\omega}{m})} = \frac{F_0}{m}\frac{m}{b\omega} = \frac{F_0}{b\omega}$

① when b=small, A is large
② when b=0, A → ∞
③ when b=large. A is small

2. **Thermal expansion: (15%)** A material is known to have the coefficient of linear expansion α at certain temperature change ΔT, prove that the volume coefficient of expansion $\beta=3\alpha$.

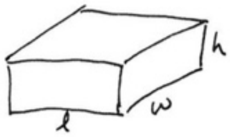
— Linear Expansion

$$\Delta L = L\alpha\,\Delta T$$

$\alpha \equiv$ Coefficient of linear expansion $[\,1/\text{deg},\ \frac{1}{K}\,]$

— Volume expansion

$$\Delta V = V\beta\Delta T \quad . \quad \beta = 3\alpha$$



at T_i $\quad V_i \equiv lwh$

$T \to T_i + \Delta T$

$V \to V_i + \Delta V$

$$V_i + \Delta V = (l+\Delta l)(w+\Delta w)(h+\Delta h)$$
$$= (l + \alpha l\Delta T)(w + \alpha w\Delta T)(h + \alpha h\Delta T)$$
$$= lwh(1+\alpha\Delta T)^3$$
$$= V_i\left[1 + 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3\right]$$

$$\therefore \frac{\Delta V}{V_i} = 3\alpha\Delta T + 3(\alpha\Delta T)^2 + (\alpha\Delta T)^3$$

neglect higher order terms

$$\frac{\Delta V}{V_i} = 3\alpha\,\Delta T$$

$$\therefore \beta = 3\alpha$$

3. <u>Doppler Effect</u>: (10%) In a general case, the frequency shift caused by the relative motion of the sound source and sound detector is described by Doppler Effect. Let f be the original sound frequency, and ***f'*** is the detected frequency due to relative motion. The speed of the sound is V and the speed of the detector is $\boldsymbol{V_D}$, the speed of the source is $\boldsymbol{V_s}$. In general, the detected frequency can be expressed as $f' = f\frac{V \pm V_D}{V \mp V_s}$. If we define the relative velocity of the wave source and the detector as $u = V_s \pm V_D$. Show that this expression can also be reduced to $f' = f\left(1 \pm \frac{u}{V}\right)$.

$$f' = f\frac{V \pm V_D}{V \mp V_s} = f(V \pm V_D)(V \mp V_s)^{-1}$$

$$= f\left(1 \pm \frac{V_D}{V}\right)\left(1 \mp \frac{V_s}{V}\right)^{-1}$$

$$= f\left[1 \pm \frac{V_D}{V} + \cdots\right]\left[1 \pm \frac{V_s}{V} + \cdots\right] \qquad u = V_s \pm V_D$$

$$= f\left(1 \pm \frac{V_s}{V} \pm \frac{V_D}{V} + \frac{V_s V_D}{V} + \cdots\right)$$

$$= f\left(1 \pm \frac{V_s \pm V_D}{V} + \cdots\right) \qquad \therefore \underline{f' = f\left(1 \pm \frac{u}{V}\right)}$$

4. **<u>Simple Harmonic Oscillator</u>**: (10%) Take a spring of force constant $\boldsymbol{k}$, with a mass $\boldsymbol{m}$ attached to the end. This oscillator is allowed to oscillate freely with maximum amplitude A. Prove that the total energy of a simple harmonic oscillator is constant. (b) **<u>Simple harmonic Oscillation:</u>** A spring mass oscillator has a total energy $\boldsymbol{E_0}$ and an amplitude of $\boldsymbol{x_0}$. Let the spring constant is $\boldsymbol{k}$. (a) How large will the kinetic energy ($\boldsymbol{E_k}$) and potential energy ($\boldsymbol{E_p}$) be for it when $x=\boldsymbol{x_0}/2$? (b) For what value of $\boldsymbol{x}$ will $\boldsymbol{E_k} = \boldsymbol{E_p}$?

Let ~~x(t)~~ $x(t) = A\cos(\omega t+\phi)$ represents a SHO's Amplitud

then $v = \frac{dx(t)}{dt} = -\omega A\sin(\omega t+\phi)$

Total energy of a SHO $E_{tot} = E_K + E_u$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2A^2\sin^2(\omega t+\phi)$$

$$E_U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$E_{total} = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi) + \frac{1}{2}kA^2\cos^2(\omega t+\phi)$$

$$= \frac{1}{2}kA^2 = \text{Constant}.$$

E_{total} determined by k and Max. Amplitude A

4-2

4. The total energy is $E_0 = E_k + E_p$

or, $\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + E_K$, Let the spring constant be k

(a) When $x = \frac{1}{2}x_0$, $E_p = \frac{1}{2}k(\frac{1}{2}x_0)^2$

$$= \frac{1}{4}E_0$$

$$E_K = E_0 - E_p$$
$$= \frac{1}{2}kx_0^2 - \frac{1}{4}E_0$$
$$= E_0 - \frac{1}{4}E_0$$
$$= \frac{3}{4}E_0$$

(b) If $E_p = E_k$ @ $E_p = \frac{1}{2}E_0$

$$= \frac{1}{2}\,\frac{1}{2}kx_0^2$$
$$= \frac{1}{4}kx_0^2$$
$$= \frac{1}{2}kx^2$$

$$\therefore x = \frac{x_0}{\sqrt{2}}$$

5.

Solution: ***(Set-up from the basic)***

$(a)\bullet$ •••••• ••••••••••••••••••••• $F = \dfrac{GmM_{ins}}{R^{\bullet}}$ ••••

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$$M_{ins} = \rho V_{ins} = \rho \frac{\bullet \pi R^{\bullet}}{\bullet} \ \bullet\bullet\bullet\bullet\bullet\bullet\bullet \because \bullet V_{ins} = \frac{\bullet \pi R^{\bullet}}{\bullet}$$

$$So \bullet F = \frac{\bullet \pi G m \rho}{\bullet} R = kR \bullet\bullet \ \bullet\bullet\bullet\bullet k = \frac{\bullet \pi G m \rho}{\bullet} = \bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$$

• ••••••••••• •••••• $\bullet F = -kx$

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$$f = \bullet \pi \sqrt{\frac{k}{m}} \ \bullet\bullet\bullet\bullet$$

$$\bullet\bullet\bullet\bullet T = \frac{\bullet}{f} = \frac{\bullet}{\bullet \pi}\sqrt{\frac{m}{k}} = \frac{\bullet}{\bullet \pi}\sqrt{\frac{m}{\frac{\bullet \pi G m \rho}{\bullet}}} = \sqrt{\frac{\bullet}{\bullet \pi G \rho}}$$

6.

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$\bullet_{.} = A\bullet\bullet\bullet\bullet kx - \omega t\bullet$••••• ••••• • ••

$\bullet_{.} = A\bullet\bullet\bullet\bullet kx + \omega t\bullet$•••• ••••••

$\bullet b\bullet\bullet$ •••••••••••••••••••••••••••••••••••••• •••• ••••• ••

•••••••• • •••$_{.} + y_{.}$

•••••••••• $\bullet A\bullet\bullet\bullet\bullet kx - \omega t\bullet\bullet + A\bullet\bullet\bullet\bullet kx + \omega t\bullet$

•••••••••• $\bullet A\bullet\bullet\bullet\bullet\bullet kx - \omega t\bullet + \bullet\bullet\bullet\bullet kx + \omega t\bullet\bullet$

•••••••••• •• $A\bullet\bullet\bullet\ kx\bullet\bullet\bullet\ \omega t$•••••••• •• •••••• $C + \bullet\bullet\bullet\ D = \bullet\ \bullet\bullet\ \frac{C+D}{\bullet}\bullet\bullet\bullet\frac{D-C}{\bullet}$

$\bullet c\bullet\bullet$ •• $A\bullet\bullet\ kx = \bullet$

• •••••• $kx = \bullet$ •••• $\bullet\bullet\bullet\bullet kx = n\pi$

•••••••• $\frac{\bullet\,\pi x}{\lambda} = n\pi$•••••••••• $\bullet\bullet\bullet\bullet x = \frac{n}{\bullet}\lambda$•••• •••• $\bullet n = \bullet$ ••••• ••••••••••••$\bullet N$

$So\bullet\bullet\bullet x = \bullet\bullet\frac{\lambda}{\bullet}\bullet\lambda\bullet\frac{\bullet\lambda}{\bullet}$•• • • •