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General Physics I, Final 1
PHYS1000AA, AB, AC, Class year 113-1
01-02-2025

SN:____________________, Name:____________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are NOT allowed.

Problems (5 Problems, total 100%)

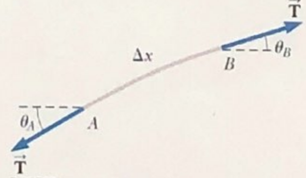
1. **Forced oscillation:** In a forced oscillation, if you applied a force $F_0 \mathrm{Sin}\omega t$ to the oscillation, where the displacement of the oscillation can be represented as $x(t) = A\cos(\omega t + \phi)$, after applied the force, the amplitude A can be calculated as $A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2+(\frac{b\omega}{m})^2}}$. (a) Draw the amplitude as function of the applied frequency ω is near the natural frequency and when the damping is very large, small and b=0. (b) Briefly explain your answer in a,

2. **Standing wave (20%):** In an area confined by two walls, similar to the block in the last problem confined between two walls. If you send a right traveling wave from the left wall and a left traveling wave from the right wall, both waves are identical except the traveling direction. You could generate a standing wave. (a) What is the possible wave function of this standing wave? (b) Why it is a standing wave?

3. **Doppler effect:** (15%) If a sound wave has a speed v and frequency f. What is the detected frequency when the source is moving at speed v_s towards the detector and the detector is stationary? (10%) Derive this.

4. **Simple harmonic Oscillation:** A spring mass oscillator has a total energy $\boldsymbol{E_0}$ and an amplitude of $\boldsymbol{x_0}$. Let the spring constant is $\boldsymbol{k}$. (a) How large will the kinetic energy ($\boldsymbol{E_k}$) and potential energy ($\boldsymbol{E_p}$) be for it when $x=\boldsymbol{x_0}/2$? (b) For what value of $\boldsymbol{x}$ will $\boldsymbol{E_k} = \boldsymbol{E_p}$?

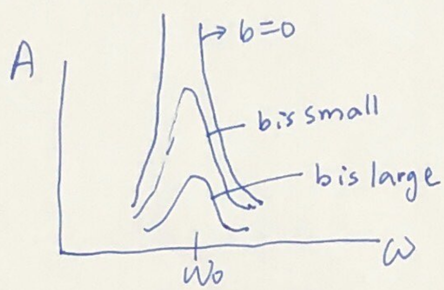
5. **Wave Equation**: Use the figure to the right to derive the wave equation for a string. If you focused on this section of the string, you can find the mass of the string is oscillating vertically (y-direction) that is it is perpendicular to the wave's travelling direction (say, to the right or in the +x direction). Let the same section, suppose the vibration of the string can be represented as a function ***y(x, t)***; a function of both ***x*** and ***t***. Prove (or derive) that the wave equation describing this wave motion is $\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}$, where $\boldsymbol{\mu}$ is the mass density and $\boldsymbol{T}$ is the tension in the string.



1. (a) $x = A\cos(\omega t + \phi)$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

When the applied oscillation is near ω_0.
depending on the value of the damping ~~tea~~ term b.
there are 3 posibilities. b = small, b is large. b = 0



(b) When ω is near the natural frequency ω_0
the amplitude depends on the damping term
$\left(\frac{b\omega}{m}\right)^2$, $A = \frac{F_0/m}{\left(\frac{b\omega}{m}\right)} = \frac{F_0}{m}\frac{m}{b\omega} = \frac{F_0}{b\omega}$

① when b = small, A is large
② when b = 0, $A \to \infty$
③ when b = large. A is small

2. **Solution:**

Let $y_1 = A\sin(kx - \omega t)$ traveling to the right

$y_2 = A\sin(kx + \omega t)$ traveling to the left

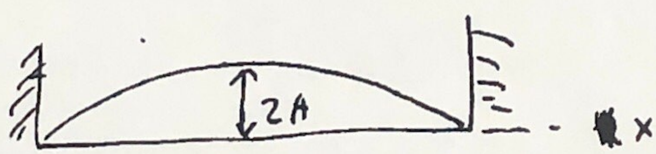
The resulting wave is

(a) $$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$= \underline{2A\sin(kx)\cos(\omega t)}$$

(b) You have a new wave

$$y = \underbrace{2A\sin(kx)}_{\text{Amplitude}}\ \underbrace{\cos(\omega t)}_{\text{Oscillation}}$$



the wave form oscillates as a function of $\cos(\omega t)$. it is not traveling anymore it stands there. So it is called Standing Wave.

3

$S \xrightarrow[V_s]{})))\to v$ • Detector

When Source moves towards the detector @ speed V_s, Wave front moves vT during a period of time T, The source moves V_sT during the same time period of T, (w

the distance between W_1 and W_2 is the detected λ' for the detector.

$$\lambda' = vT - V_sT$$

$$\therefore f' = \frac{v}{\lambda'} = \frac{v}{vT - V_sT} = \frac{v}{\frac{v}{f} - \frac{V_s}{f}} = f\,\frac{v}{v - V_s}$$

4. The total energy is $E_0 = E_k + E_p$

or, $\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + E_K$, Let the spring constant be k

(a) When $x = \frac{1}{2}x_0$, $E_p = \frac{1}{2}k(\frac{1}{2}x_0)^2$

$$= \frac{1}{4}E_0$$

$$E_K = E_0 - E_p$$

$$= \frac{1}{2}kx_0^2 - \frac{1}{4}E_0$$

$$= E_0 - \frac{1}{4}E_0$$

$$= \frac{3}{4}E_0$$

(b) If $E_p = E_K$ @ $E_p = \frac{1}{2}E_0$

$$= \frac{1}{2}\,\frac{1}{2}kx_0^2$$

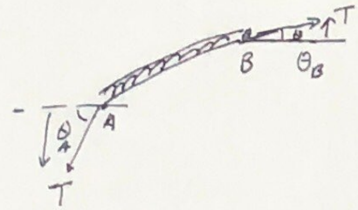
$$= \frac{1}{4}kx_0^2$$

$$= \frac{1}{2}kx^2$$

$\therefore x = \frac{x_0}{\sqrt{2}}$

5.

Net force in $\hat{y}$ direction



$$\sum F_y = T\sin\theta_B - T\sin\theta_A$$

$$= T(\sin\theta_B - \sin\theta_A) \quad \text{if } \theta \text{ is small, } \sin\theta \approx \tan\theta$$

$$\approx T(\tan\theta_B - \tan\theta_A)$$

$$= T\left(\frac{\partial y}{\partial x}\Big|_{\text{at }B} - \frac{\partial y}{\partial x}\Big|_{A}\right) = m a_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\therefore \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\text{or} \quad \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - (\frac{\partial y}{\partial x})_A}{\Delta x}$$

$$= \frac{\partial^2 y}{\partial x^2} \qquad \text{Note. } \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\therefore \boxed{\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}} \quad \text{or} \quad \boxed{\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{-\omega^2\mu}{T}\sin(kx - \omega t) = -k^2\sin(kx - \omega t)$$

$$\rightarrow k^2 = \frac{\mu}{T}\omega^2, \quad v = \frac{\omega}{k}$$

$$\therefore v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu} \quad \rightarrow \boxed{v = \sqrt{\frac{T}{\mu}}}$$