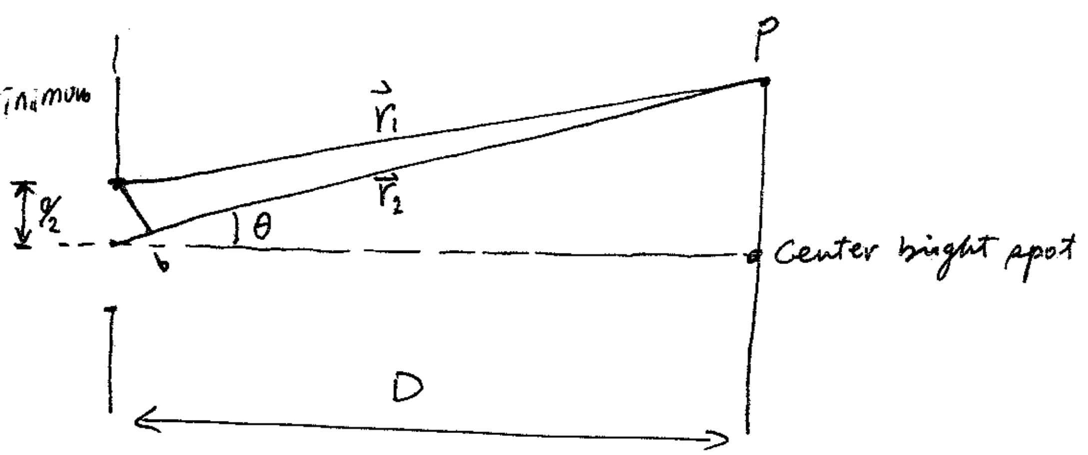
1. Diffraction

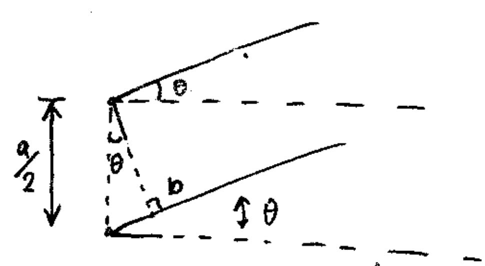
- Flaring of light emerging from a narrow slit.
- Light produces an interference pattern due to diffracted light. Called diffraction pattern, check Fig 37-1 and Fig. 37-2.
- Diffraction can NOT explained by using geometric optics, since light travels in straight line.

2. Fresnel's theory – Using wave theory to explain diffraction.

① Diffraction by a single slit of width a

(1) 1st minimum





Assume $D >> a$.
$\vec{r}_1 \parallel \vec{r}_2$

- Waves from different points (between r_1 and r_2) reaching the viewing screen C, ~~ung~~ undergoes interference and produce a diffraction pattern on the screen.
- for Center point, all waves reaching the center point travel ~~at~~ about the same distance.
- First minimum occurs at

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}. \quad \rightarrow \quad a\sin\theta = \lambda.$$

$$\text{or } \sin\theta = \frac{\lambda}{a}$$

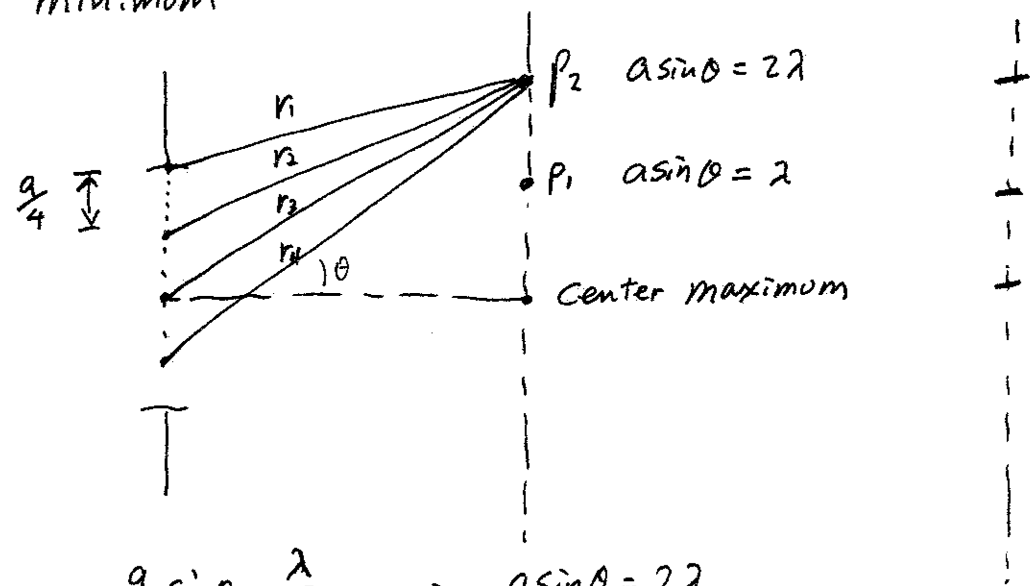
- The smaller the slit width a, the bigger the angle of the diffracted light. ∴ More flaring occurs for smaller slit

— if $a \approx \lambda$. $\rightarrow$ $a \sin\theta = \lambda$

$\sin\theta = 1$. $\theta = 90°$

Therefore if the slit width $\approx$ the wave length. The first minimum occurs at the edges of the viewing screen, therefore the center bright region will cover the entire screen.

(2) 2^{nd} minimum

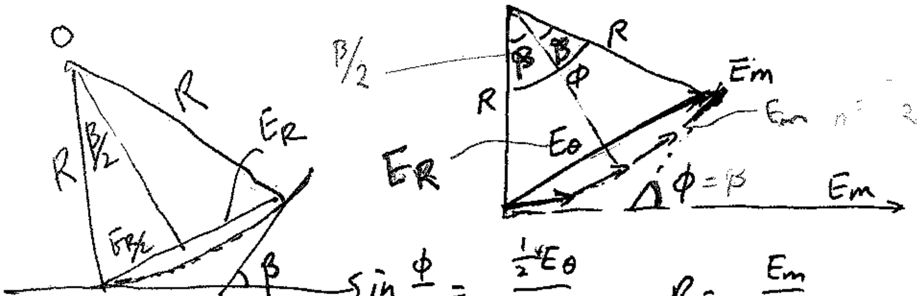


$$\frac{a}{4}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = 2\lambda$$

(3) n^{th} minimum $\rightarrow$ $a\sin\theta = n\lambda$ (n^{th} dark fringes)

$n = 1, 2, 3, 4 \cdots$

(2) The intensity: if we divide the slit into infinitesimal zones of Δx, the phasor diagram look like



$\alpha = \frac{\phi}{2}$, $\phi = \frac{E_m}{R}$

E_m = the amplitude at the center diffraction pattern = the arc

E_θ = Amplitude of the resultant wave at P. with angle θ

$$\sin\frac{\phi}{2} = \frac{\frac{1}{2}E_\theta}{R}, \quad R = \frac{E_m}{\phi}$$

$$\therefore \sin\frac{\phi}{2} = \frac{E_\theta \phi}{2E_m} \longrightarrow E_\theta = \frac{E_m}{\frac{1}{2}\phi}\sin\frac{\phi}{2}$$

ϕ = total phase difference between E_m's

$$I \propto |E_\theta|^2$$

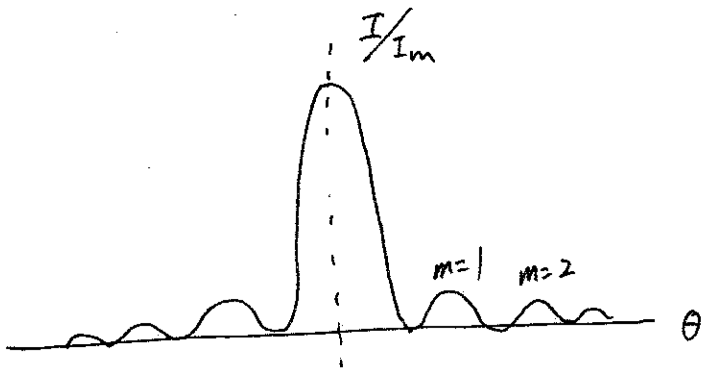
$$\frac{I}{I_m} = \frac{E_\theta^2}{E_m^2} \quad \therefore I = I_m\left(\frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}}\right)^2$$

$$\frac{\phi}{2\pi} = \frac{a\sin\theta}{\lambda} \rightarrow \phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta)$$

$$\therefore \quad I = I_m \left(\frac{\sin \frac{\phi}{2}}{\frac{\phi}{2}} \right)^2 \qquad \text{if } \frac{\phi}{2} = \alpha$$

$$\boxed{\begin{gathered} I = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2 \\ \phi = \frac{2\pi}{\lambda} a \sin \theta \end{gathered}}$$

ϕ = the phase differente between the top and bottom of the two rays in the slit



→ Refer to Fig. 37-7

① Minimom occurs at $\alpha = m\pi$

$$\therefore \quad m\pi = \frac{a\pi}{\lambda} \sin\theta \qquad m = 1, 2, 3 \quad \ldots \text{minimum}$$

$$\boxed{a \sin\theta = m\lambda} \quad - \text{minimum}$$

② 2^{nd} max $\quad \alpha = (m + \frac{1}{2})\pi$

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha} \right)^2 = \left(\frac{\sin (m+\frac{1}{2})\pi}{(m+\frac{1}{2})\pi} \right)^2, \quad m = 1, 2, 3$$

$m = 1$,

$$\frac{I_1}{I_m} = \left(\frac{\sin (1+\frac{1}{2})\pi}{(1+\frac{1}{2})\pi} \right)^2 \approx 4.5\%$$

$m = 2 \quad \frac{I_2}{I_m} = 1.6\%$

$m = 3 \quad \frac{I_3}{I_m} = 0.83\%$

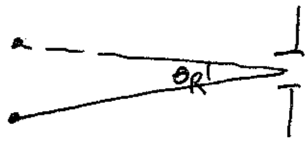
3. Diffraction by a circular aperture

— for a distant pt source, such as a star, the image is not a point, rather, it's a circular disk surrounded by several secondary rings a diffraction pattern through the aperture of a converging lens

$\cancel{d}\sin\theta = 1.22\frac{\lambda}{d}$ (first minimum)

$a\sin\theta = \lambda$ (first minimum of a single slit)

— Resolvability – the ability to resolve two object distant from the aperture of an optics when the angular separation is small i.e. $\theta \approx 0$



$\boxed{\text{Refer to Fig 37-10}}$ page. 938

Rayleigh's criterion — Two objects that are barely resolved must have an angular separation

λ = wavelength, d = diameter of lens

$$\theta_R = \sin^{-1}\frac{1.22\lambda}{d}$$

$\sin\theta \approx \theta$ when $\theta \approx 0$

$$\boxed{\theta_R = 1.22\frac{\lambda}{d}}$$

— only an approximation resolvability depends on many factors.
* relative brightness
* surroundings
* turbulence in the air

Increase the resolvability.
— Increasing the lens diameter
— Using shorter wavelength (UV)

4. Diffraction by a double slit

— In Young's experiments, One assumed $a \ll \lambda$

— ~~In~~ diffraction by a larger slit $a > \lambda$

— double slit combine the above two situations

— diffraction acts like a ~~...~~ envelope

Therefore, by taking into account of the finite size of the aparture,

$$I = I_m (\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$\beta = \left(\frac{\pi d}{\lambda}\right)\sin\theta$,   d = distance between slits

$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta$,   a = width of the slits.

① if $a \approx 0$, then $\alpha \approx 0$ $\frac{\sin\alpha}{\alpha} \approx 1$

$I = I_m(\cos^2\beta)$ — interference only from two narrow slits.

② if $d \approx 0$, then $\beta \approx 0$, $\cos^2\beta = 1$

$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$ — diffraction.

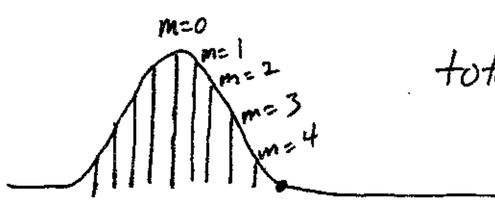
$\boxed{\text{Example : 37-5}}$

$\lambda = 405\,nm$
$d = 19.44\,\mu m$ – slits separations
$a = 4.050\,\mu m$ – slit width

① $a\sin\theta = \lambda$ — diffraction 1st minimum

$d\sin\theta = m\lambda$ — interference bright fringes (double slit)

$m = \frac{d}{a} = \frac{19.44}{4.05} = 4.8$



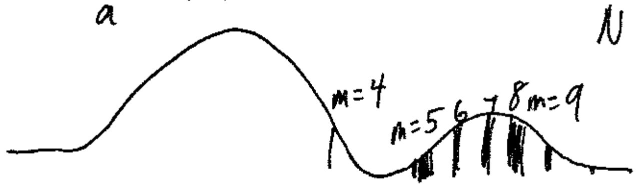
total N = 9 fringes within the central maximum.

② $a\sin\theta = 2\lambda$

$d\sin\theta = m\lambda$

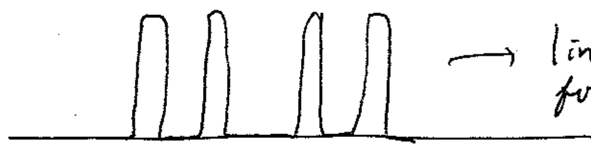
$m' = \frac{2d}{a} = 9.6$

$N' = 5$



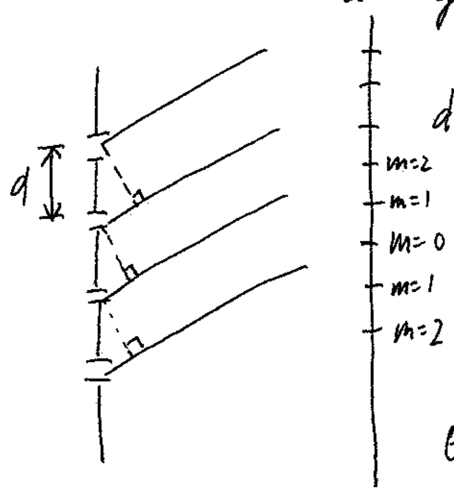
5 Diffraction grating — useful tool to study light.

— great number of slits. called ruling, N slits (1000/mm)



→ lines on a viewing screen for a monochromatic light incidence.

— Consider this interference similar to that of a double slit,
But $d = \frac{w}{N}$, w = width of the whole grating
N = ruling of the grating,
d = grating spacing



$d \sin\theta = m\lambda \quad m = 0, 1, 2, 3 \ldots$

for maximum lines

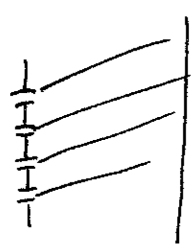
m's are called order numbers

$m=0$ → first order

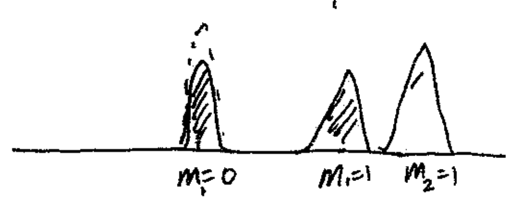
$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$ $\quad m=1,\ \theta = ?$
$m=2,\ \theta = ?$
θ is big between ord

— therefore from the order, one can decide the wave length of an unknow light,

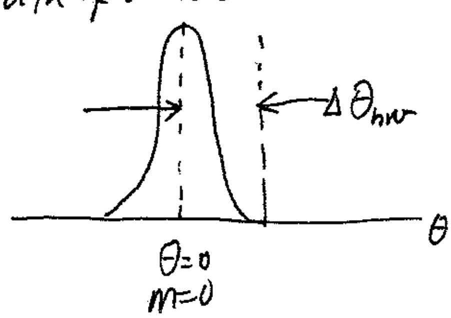
— The grating's ability to resolve (separate) lines of different wave lengths depends on the width of the lines.



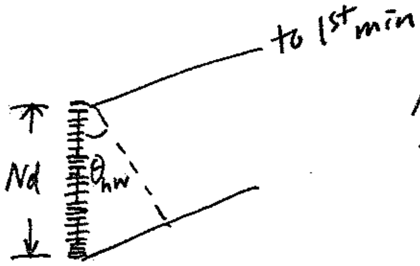
1st order



— Width of the line

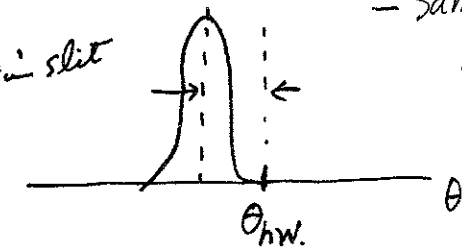


, The whole width of the center line is $2\Delta\theta_{hw}$



$Nd \sin\Delta\theta_{hw} = \lambda$
— top and bottom rays have complete cancellation.
— condition for first minimum.
— Same as single slit diffraction

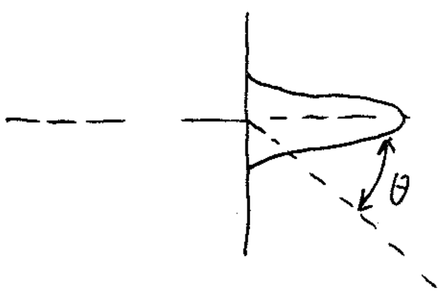
all slits add together ~~big~~ like a diffraction slit



$Nd \sin\Delta\theta_{hw} = \lambda$.

if $\Delta\theta_{hw}$ is small, $\sin\Delta\theta_{hw} \approx \Delta\theta_{hw}$.

$\therefore \boxed{\Delta\theta_{HW} = \frac{\lambda}{Nd}}$ — line width for zeroth order ($m=0$) of a N ruling grating.



$\Delta\theta_{HW} = \frac{\lambda}{Nd\cos\theta}$ for other lines.

— $\Delta\theta_{hw}$ decreases when N is larger
— better resolution when d is ~~smaller~~ larger.
— resolution is better for larger N

refer to p. 944 for hydrogen lines

6. Dispersion — the spreading of the ~~gra~~ different colors by a grating

$$D \equiv \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$$

$\Delta\theta$ = angular separation
$\Delta\lambda$ = wavelength difference

$d\sin\theta = m\lambda$
$d\cos\theta\, d\theta = m\, d\lambda$
$d\cos\theta\, \Delta\theta = m\Delta\lambda$
$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}$

7. Resolving power

$$R \equiv \frac{\lambda_{av.}}{\Delta\lambda}$$

the bigger the number, the better the resolving power.

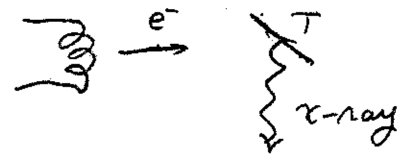
$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta}$

But $d\cos\theta\,\Delta\theta = m\Delta\lambda$
$d\cos\theta\,\Delta\theta_{hw} = m\Delta\lambda = \frac{\lambda}{N}$

$$\therefore R = \frac{\lambda}{\Delta\lambda} = Nm$$

8. X-ray diffraction

X-ray ; $\lambda \approx 10^{-10} m = 1 \mathring{A}$



$\lambda_{visible} = 550 nm$.
$= 5.5 \times 10^{-7} m$

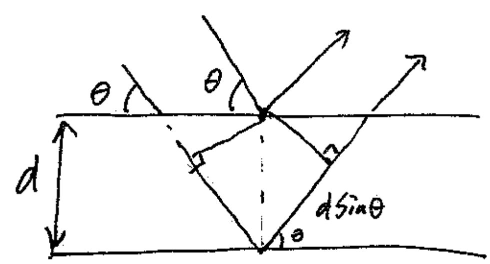
— Standard grating $\Delta\theta_{HW} = \frac{\lambda}{Nd\cos\theta}$ But for $\lambda = 10^{-10} m$
$d = 3000 nm$

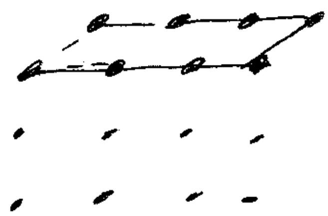
$\theta = \sin^{-1}\frac{m\lambda}{d} = \sin^{-1}\frac{(1)(0.1 nm)}{3000 nm} \approx 0.0019°$

too small to be distiguished and observed.

— Natural crystalline consists of a regular array of atoms, forms a natural 3-D grating, as $d \approx 1 \mathring{A}$.

— discovered by German physicist. Max Von Laue (191[illegible]



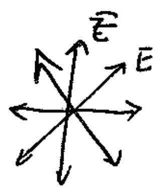


Note: θ is defined differently.

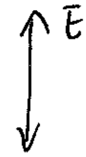
$2d\sin\theta = m\lambda$ — bright spots
— for $m = 1, 2, 3, \cdots$
— Bragg's law, British physicist W. L. Bragg
— θ = Bragg's angle

38-6 polarization of light wave.

— polarization of EM wave : defined as the direction of the electric field is vibrating



unpolarized light — the direction of propagation of the light is ~~polarized~~ perpendicular to the page.



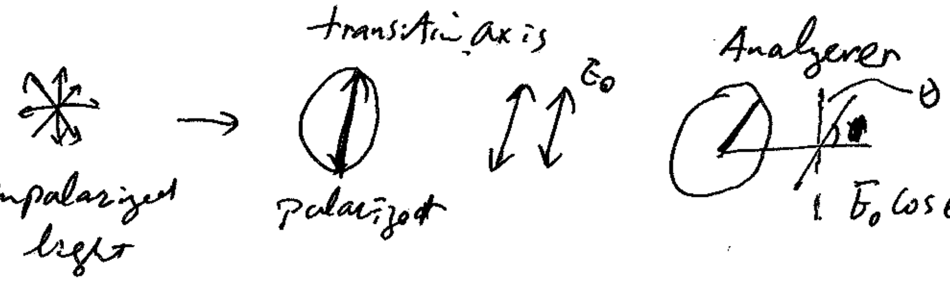
→ polarized light.

→ linearly polarized = the resultant electric field $\vec{E}$ vibrates in the same direction at all time at a particular point.

→ plane-polarized

→ polarized

Polarization by selective absorption

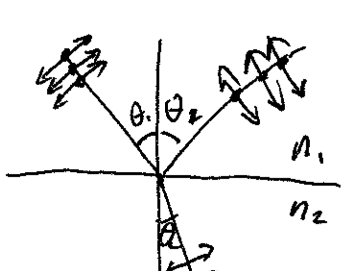


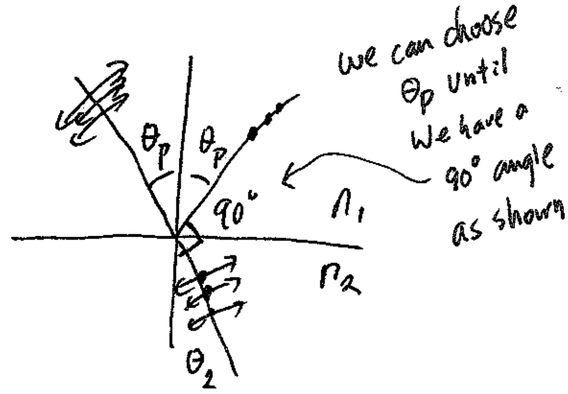
→ the polarized beam transmitted through the analyzer

$$I = I_{max} \cos^2\theta \quad \text{Malus's law}$$

Polarization by Reflection

— Unpolarized light reflected from a surface may be polarized





the angle between the reflected and refracted light is 90° → the reflected beam is completely polarized
the refracted beam is still partially polarized
θ_p is called polarizing angle θ_p

$$\theta_p + 90^\circ + \theta_2 = 180^\circ$$

$$\therefore \theta_2 = 90^\circ - \theta_p$$

Using Snell's law, if $n_1 = 1$ for air
$n_2 = n$.

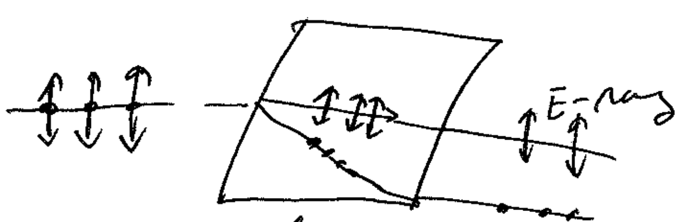
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n \sin\theta_2$$

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\sin(90^\circ - \theta_p)} = \frac{\sin\theta_p}{\cos\theta_p}$$

$n = \tan\theta_p$ — Brewster's law
$\theta_p \equiv$ Brewster's angle

Page 1228.
Fig 38.32 — Brewster angle is a function of ~~angle~~ wave length

Polarization by double refraction



O-ray

O. and E are mutually perpendicular to each other

double refracting or birefrigent materials → the speed of light is not the same in all directions
→ One ray is called Ordinary ray
The other is call extra Ordinary ray

① Fig 38-37

② Blue sky. $d \ll \lambda$
~~Scattering~~ Scattered Intensity $I_s \propto \frac{1}{\lambda^4}$