Chap 34 Electromagnetic Wave

34.1 Maxwell's Equations + Hertz discoveries

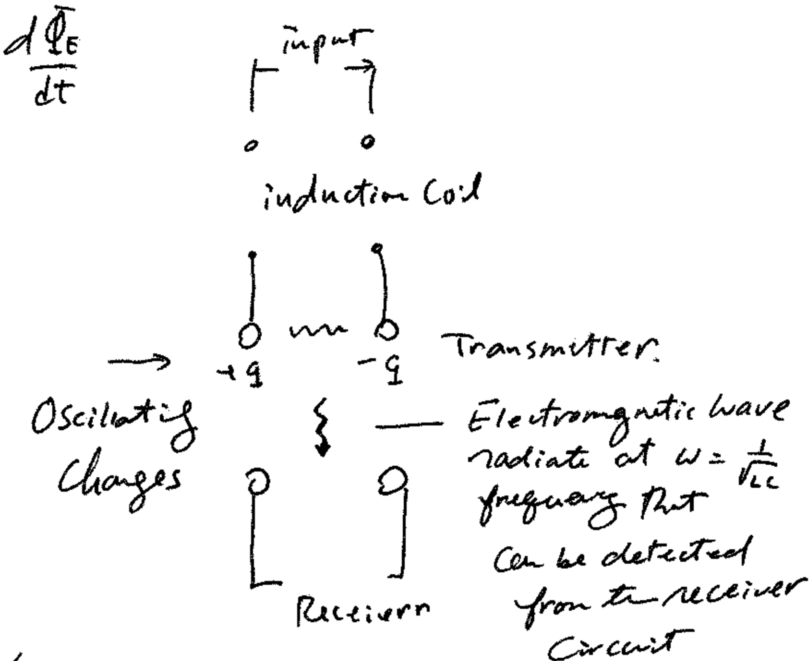
$$\oint E \cdot dA = \frac{q}{\epsilon_0}$$

$$\oint B \cdot dA = 0$$

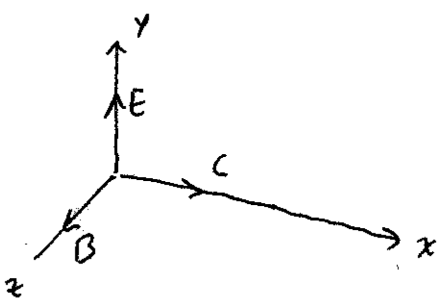
$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$

$$\oint B \cdot ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Hertz experiment:



34.2 Plane Electromagnetic wave



① E. B are linearly polarized wave

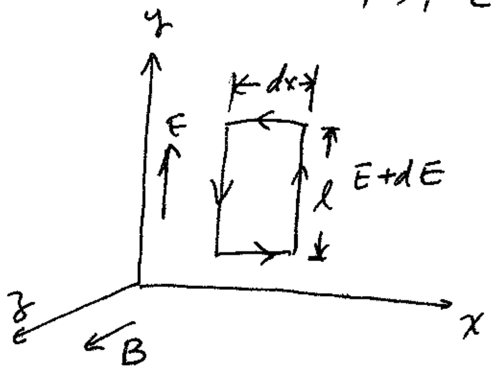
② Wave can radiate from any position in the yz plane
→ the entire collection of the rays are called plane wave

③ A point Source will produce a spherical wave.

④ In empty space $q=0$, $I=0$

$$\oint B \cdot ds = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$



$$\oint E \cdot ds = [E(x+dx,t)]l - [E(x,t)]l$$

$$= \left[E(x,t) + \frac{dE}{dx}\Big|_{t=\text{const}} dx\right]l - [E(x,t)]l$$

$$= \left[E(x,t) + \frac{\partial E}{\partial x} dx\right]l - [E(x,t)]l$$

$$\simeq l\left(\frac{\partial E}{\partial x}\right)dx$$

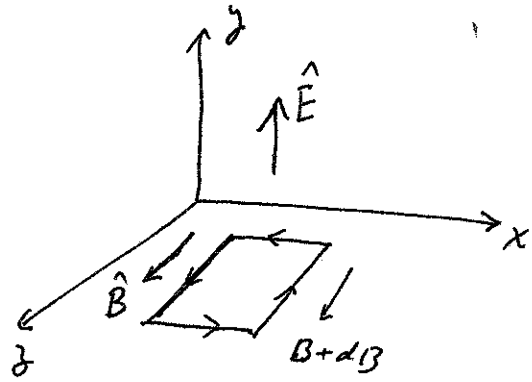
$$\Phi_B = Bl\,dx$$

$$\frac{d\Phi_B}{dt} = l\,dx\left(\frac{dB}{dt}\right)_{x=\text{const}} = l\,dx\frac{\partial B}{\partial t}$$

$$\therefore \quad l\left(\frac{\partial E}{\partial x}\right)dx = -l\,dx\frac{\partial B}{\partial t}$$

$$\boxed{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}} \quad - (34.6)$$

Similarly:



$$\oint B \cdot ds = [B(x,t)]l - [B(x+dx,t)]l$$

$$= -l\left(\frac{\partial B}{\partial x}\right)dx$$

$$\Phi_E = El\,dx$$

$$\frac{d\Phi_E}{dt} = l\,dx\frac{\partial E}{\partial t}$$

$$\rightarrow -l\left(\frac{\partial B}{\partial x}\right)dx = \mu_0\epsilon_0\,l\,dx\left(\frac{\partial E}{\partial t}\right)$$

$$\therefore \quad \boxed{\frac{\partial B}{\partial x} = -\mu_0\epsilon_0\frac{\partial E}{\partial t}} \quad - (34.6)$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0\frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}} \quad -(34.8)$$

Similarly:

$$\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\mu_0\epsilon_0\frac{\partial E}{\partial t}\right) = -\mu_0\epsilon_0\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial t}\right) = -\mu_0\epsilon_0\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial x}\right)$$

$$= -\mu_0\epsilon_0\frac{\partial}{\partial t}\left[-\frac{\partial B}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}} \quad -(34.9)$$

$\rightarrow \quad c = \dfrac{1}{\sqrt{\epsilon_0\mu_0}}$ $\rightarrow$ the speed of the EM wave

Solutions of (34.8) (34.9)

$$E = E_{max}\cos(kx - \omega t)$$
$$B = B_{max}\cos(kx - \omega t)$$

$$\omega = 2\pi f$$
$$k = \frac{2\pi}{\lambda}$$
$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

$$\frac{\partial E}{\partial x} = -k E_{max}\sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{max}\sin(kx - \omega t)$$

$$\rightarrow \quad k E_{max} = \omega B_{max}$$

$$\frac{E_{max}}{B_{max}} = \frac{\omega}{k} = c$$

① EM wave obeys the superposition Principle

② the Solutions of Maxwell's equation are wave like

③ EM has speed in Vacuum $c = 1/\sqrt{\mu_0\epsilon_0}$

④ E and B are perpendicular to each other and perpendicular to the direction of wave propagation

⑤ $E/B = c$

34.3 Energy Carried by EM wave

the rate of flow of energy defined as $\vec{S} \equiv \frac{1}{\mu_0} \vec{E} \times \vec{B}$

, the poynting vector $\vec{S}$.
- rate of flow of energy through a surface area perpendicular to the direction of wave propagation.
- power per unit area.

$[J/s \cdot m^2] = W/m^2$

for a plane wave $|E \times B| = EB$. $\therefore S = \frac{1}{\mu_0} EB$

$$S = \frac{1}{\mu_0} EB. \quad \text{But} \quad \frac{E}{B} = c$$

$$= \frac{1}{\mu_0} Bc \cdot B$$

$$= \frac{c}{\mu_0} B^2$$ — Instantaneous rate at which energy is passing through the unit area

$$I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{max}^2$$

Recall. energy per unit volume $\quad u_E = \frac{1}{2}\epsilon_0 E^2$, $\quad u_B = \frac{1}{2\mu_0} B^2 \quad (26.13)$

for an EM wave

$$u_B = \frac{1}{2\mu_0}\left(\frac{E}{c}\right)^2 = \frac{\epsilon_0 \mu_0}{2\mu_0} E^2 = \frac{1}{2}\epsilon_0 E^2$$

$$\rightarrow u_B = u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$ — the instantaneous energy density associated with the magnetic field of an EM wave is the same as the electric field.

Total instantaneous energy density

$$u = u_B + u_E = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$u_{av} = \epsilon_0 (E^2)_{av} = \frac{1}{2}\epsilon_0 E_{max}^2 = \frac{1}{2\mu_0} B_{max}^2$$

$\therefore I = S_{av} = c\, u_{av}$ — the intensity of an EM wave equals the average energy density multiplied by the speed of light.

34.4 Momentum and Radiation Pressure

Suppose the EM wave strikes the ~~#~~ surface perpendicularly and transports total energy U @ time Δt, if the energy were absorbed totally,

The total momentum transported : p

$$p = \frac{U}{c} \quad (\text{Check } 20.7)$$

The pressure $P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{c}\right) = \frac{1}{c}\frac{dU/dt}{A}$

$\frac{dU}{Adt}$ = the rate of the energy arriving the surface = Poynting vector

$$\therefore \; P = \frac{S}{c}$$

if the surface is a perfect reflector $p = \frac{2U}{c} \Rightarrow P = \frac{2S}{c}$

Radiation pressures are very small $\sim 5 \times 10^{-6}\ N/m^2$

34.5 Production of EM wave by an ~~Ant~~ Antena

→ Time varying charges or current emit EM radiation

→ When a charge accelerated, it radiates energy