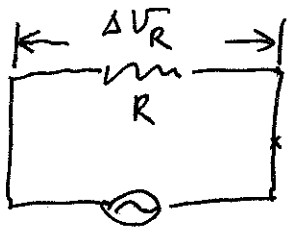
Chap 33. Alternating Current Circuits

33.1 AC Sources

Time varying voltage $\Delta V = \underbrace{\Delta V_{max}}_{\text{voltage amplitude}} \sin \omega t$ $\omega = 2\pi f = \frac{2\pi}{T}$

: generators
: electric outlet

Add a resistor

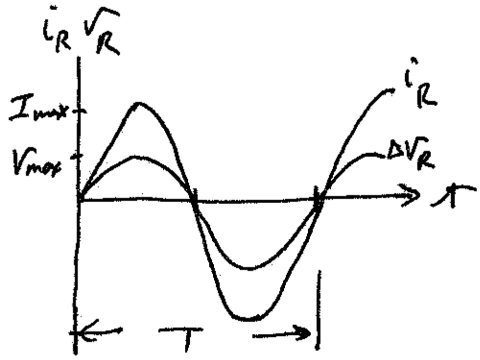


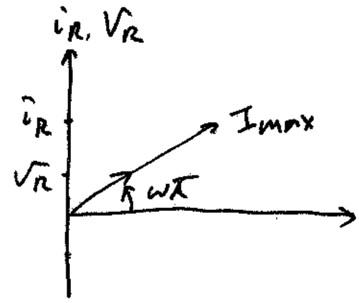
$\therefore \Delta V = \underbrace{\Delta V_R}_{\text{Instantaneous voltage across the resistor}} = \Delta V_{max} \sin \omega t$

$$i_R = \frac{\Delta V_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

$$I_{max} = \frac{\Delta V_{max}}{R}$$

or $\Delta V_R = I_{max} R \sin \omega t$





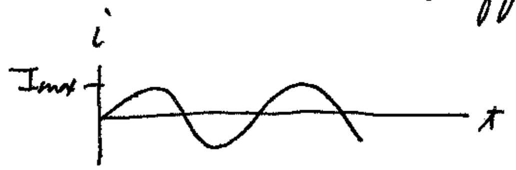
phasor diagram

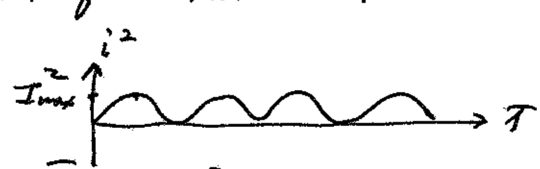
→ phasor (A vector)

→ the current in a resistor is always in phase with the applied AC voltage source.

→ the average value of the current over one cycle is zero.

Note: the direction of the current does not affect the thermal effect of the resistor.





$\overline{i^2} = \frac{1}{2} I_{max}^2$ →

the rate the energy is delivered to a resistor is the power. $P = i^2 R$ → the direction of the current does not matter

$$I_{rms} = \sqrt{\overline{i^2}}$$

$$= \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

$$i^2 = I_{max}^2 \sin^2 \omega t$$

$$\overline{i^2} = \frac{1}{T}\int I_{max}^2 \sin^2 \omega t\, dt$$

$$= I_{max}^2 \frac{1}{T}\int_0^T \sin^2 \omega t\, dt$$

$$= \frac{1}{2} I_{max}^2$$

$$P_{av} = I_{rms}^2 R$$

average power delivered to the resistor

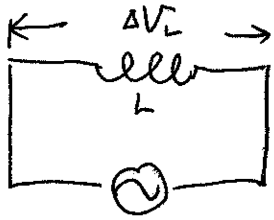
$$\frac{1}{T}\int_0^T \sin^2 \omega t = \frac{1}{T}\int_0^T \left[\frac{1-\sin\frac{\omega t}{2}}{2}\right] dT$$

$$= \frac{1}{2} - \frac{1}{T}\underbrace{\int_0^T \sin\tfrac{1}{2}\omega t\, dT}_{=0}$$

$$\Delta V_{rms} = \frac{1}{\sqrt{2}}\Delta V_{max} = 0.707\, \Delta V_{max}$$

(another way of proof refer to page 1037)

② Inductor in an AC current



$$\overbrace{\Delta V - L\frac{di}{dt}}^{\Delta V_L} = 0$$

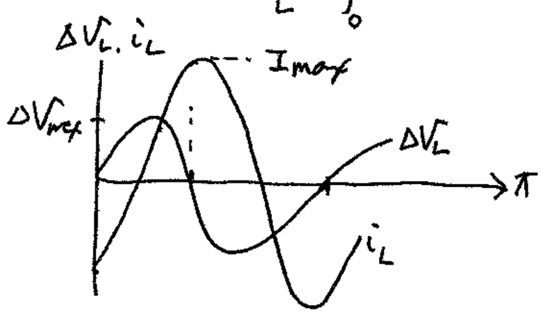
$$\Delta V = L\frac{di}{dT} = \Delta V_{max}\sin\omega t = \Delta V_L$$

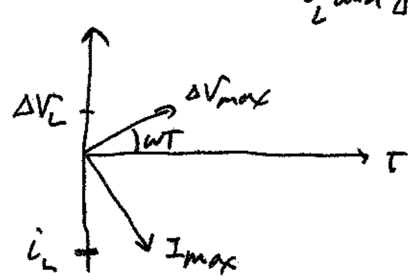
$$\frac{di}{dT} = \frac{\Delta V_{max}}{L}\sin\omega t$$

$$di_L = \frac{\Delta V_{max}}{L}\sin\omega t\, dt$$

$$i_L = \frac{\Delta V_{max}}{L}\int_0^t \sin\omega t\, dt = -\frac{\Delta V_{max}}{\omega L}\cos\omega t = \overbrace{\left(\frac{\Delta V_{max}}{\omega L}\right)}^{I_{max}}\sin\left(\omega t - \frac{\pi}{2}\right)$$

∴ i_L and ΔV_L, $\frac{\pi}{2}$ out of phase





Phasor

$$i_L = -\frac{\Delta V_{max}}{\omega L}\cos(\omega t) = \frac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

— i_L and ΔV_L is out of phase by $\frac{\pi}{2}$

— ~~leads~~ lags the AC by $\pi/2$

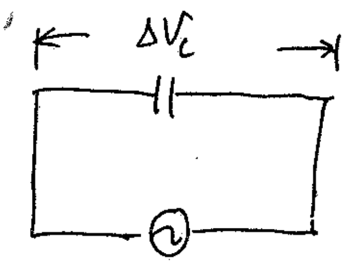
$$I_{max} = \frac{\Delta V_{max}}{\omega L}$$

$$\equiv \frac{\Delta V_{max}}{X_L}$$

$X_L \equiv$ inductive reactance

— has the same unit as resistance

③ Capacitor in an AC current



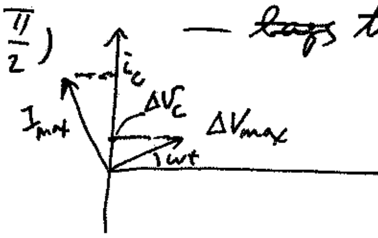
$$\Delta V_C = \Delta V = \Delta V_{max}\sin\omega t$$

$$q = C\Delta V_{max}\sin\omega t$$

$$i_C = \frac{dq}{dt} = \omega C\,\Delta V_{max}\cos\omega t$$

$$= \omega C\,\Delta V_{max}\sin\left(\omega t + \frac{\pi}{2}\right)$$

— ~~lags~~ leads the AC by $\pi/2$



$$I_{max} = \omega C\,\Delta V_{max}$$

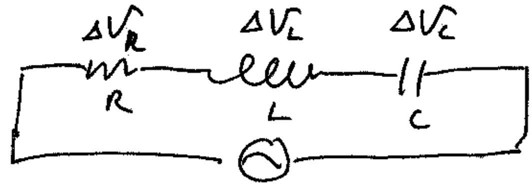
$$= \frac{\Delta V_{max}}{1/\omega C}$$

$$\equiv \frac{\Delta V_{max}}{X_C}$$

$X_C = \frac{1}{\omega C}$ = Capacitive reactance

$$\Delta V_C = \Delta V_{max}\sin\omega t = I_{max}X_C\sin\omega t$$

④ The RLC Series Circuit



1) the current at all points in a Series AC Circuit has the same amplitude and phase
— for Series circuit

$\Delta V = \Delta V_{max} \sin \omega t$

$i = i_{max} \sin(\omega t - \phi)$

ϕ = phase angle

① in general, the current will not be in phase as that of the AC voltage.
② in general. current will be the same phase

in General : $\boxed{\text{Voltage}}$

R : in phase
L : leads the current by $\frac{\pi}{2}$
C : lags the current by $\frac{\pi}{2}$

$\Delta V = \Delta V_{max} \sin \omega t$

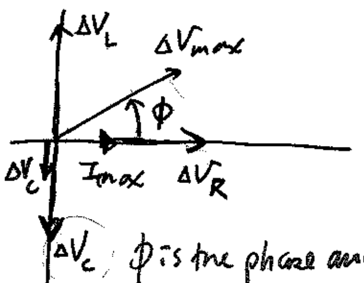
$\Delta V_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t$

$\Delta V_L = I_{max} X_L \sin(\omega t + \frac{\pi}{2}) = \Delta V_L \cos \omega t$ ← Note: this is the voltage

$\Delta V_C = I_{max} X_C \sin(\omega t - \frac{\pi}{2}) = -\Delta V_C \cos \omega t$

Also $\Delta V = \Delta V_R + \Delta V_L + \Delta V_C$

Phasor diagram (Single phasor I_{max} to represent the current in the elements)



ϕ is the phase angle between the current and applied voltage

→

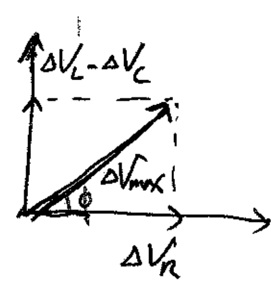
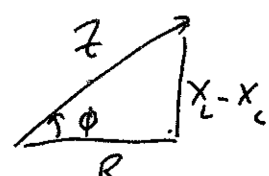


Fig 33.15 (b)

from Fig 33.15 (b) page 1045

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}$$
$$= \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$
$$= I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$
$$\equiv I_{max} Z$$



, $Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$ = Impedance Z of the circuit

① $X_L > X_C$ → more inductive than Capacitive
$X_C > X_L$ → more Capacitive than inductive
$X_L = X_C$ → pure resistive

33.6 power in an AC circuit

$$P = i\Delta V = I_{max}\sin(\omega t-\phi)\,\Delta V_{max}\sin\omega t$$ — for a RLC circuit.

$$= I_{max}\Delta V_{max}\sin\omega t\,\sin(\omega t-\phi)$$

$$= I_{max}\Delta V_{max}\sin^2\omega t\cos\phi$$

But $\sin(\omega t-\phi)$
$= \sin\omega t\cos\phi - \cos\omega t\sin\phi$

$$- I_{max}\Delta V_{max}\underbrace{\sin\omega t\cos\omega t}_{Av=0}\sin\phi$$

$\sin\omega t\cos\omega t = \frac{1}{2}\sin 2\omega t$

$$P_{av} = \overline{P} = \frac{1}{2}I_{max}\Delta V_{max}\underbrace{\cos\phi}_{\text{independent of } t}$$

$I_{rms} = \frac{I_{max}}{\sqrt{2}}$ (33.4)

$\Delta V_{rms} = \frac{1}{\sqrt{2}}\Delta V_{max}$ (33.5)

$$\therefore P_{av} = I_{rms}\,\Delta V_{rms}\underbrace{\cos\phi}_{\text{power factor}}$$

max voltage across the resistor

$= I_{max}R$

$\rightarrow \cos\phi = \frac{I_{max}R}{\Delta V_{max}}$

$$\therefore P_{av} = I_{rms}\Delta V_{rms}\cos\phi = I_{rms}\left(\frac{\Delta V_{max}}{\sqrt{2}}\right)\frac{I_{max}R}{\Delta V_{max}} = I_{rms}\frac{I_{max}R}{\sqrt{2}}$$

$$P_{av} = I_{rms}^2 R\ ,\quad I_{max} = \sqrt{2}\, I_{rms}$$

33.7 Resonance in a Series RLC Circuit

— A series RLC circuit is said to be in resonance when the current has its maximum value.

in general $I_{rms} = \frac{\Delta V_{rms}}{Z}$ $\quad Z \equiv \sqrt{R^2+(X_L-X_C)^2}$ (33.25)

$$= \frac{\Delta V_{rms}}{\sqrt{R^2+(X_L-X_C)^2}}$$

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

↑ so the impedance depends on the frequency of the source

Resonance frequency @ $X_L - X_C = 0$, $\frac{1}{\omega_0 C} = \omega_0 L$

$\rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ → Also is the same as the natural frequency in a LC circuit

→ So the current in a series RLC circuit reaches it maximum when the applied voltage match the natural frequency of the LC components

→ At resonance, if $R = 0$ then the current becomes infinite

$$P_{av} = I_{rms}^2 R = \frac{(\Delta V_{rms})^2}{Z^2} R = \frac{(\Delta V_{rms})^2 R}{R^2 + (X_L - X_C)^2}$$

But $(X_L - X_C)^2 = (\omega L - \frac{1}{\omega C})^2 = \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)^2$

with $\omega_0 = \frac{1}{LC}$

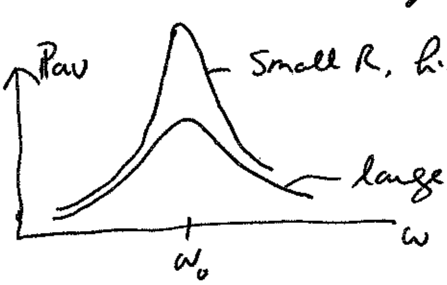
$$\therefore P_{av} = \frac{(\Delta V_{rms})^2 R \omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2}$$

at resonance, $\omega = \omega_0$

$$P_{av} = \frac{(\Delta V_{rms})^2}{R}$$

Check page 1050 Fig 33.19

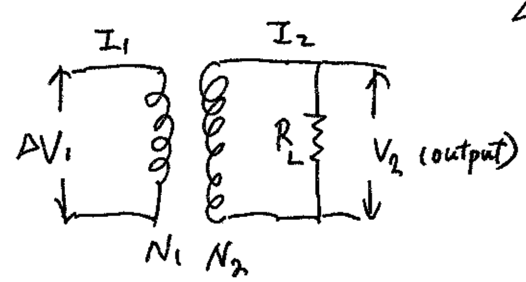
Q-factor = Quality factor $= \frac{\omega_0}{\Delta\omega}$ $\Delta\omega$ = Full width at half maximum



→ high Q circuit responses only to very narrow range of the frequency

→ The tuning of radio receiver

33.8 Transformer



$\Delta V_1 = -N_1 \frac{d\Phi_B}{dt}$. $\Delta V_2 = -N_2 \frac{d\Phi}{dt}$

$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$ or $\Delta V_2 N_1 = N_2 \Delta V_1$

$I_1 \Delta V_1 = I_2 \Delta V_2$ power is the same

flux is the same in both circuit

$I_2 = \frac{\Delta V_2}{R_L} \rightarrow I_1 = \frac{\Delta V_1}{R_{eq}}$, $R_{eq} = \left(\frac{N_1}{N_2}\right)^2 R_L$

$\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$ ① $N_2 > N_1$, Step up transformer
② $N_1 > N_2$, Step down transformer

$I_2 = \frac{\Delta V_2}{R_L}$. $I_1 = \frac{\Delta V_1}{R_{eq}}$

$$\therefore \frac{I_2}{I_1} = \frac{\Delta V_2}{R_L}\frac{R_{eq}}{\Delta V_1} = \frac{\Delta V_2}{\Delta V_1}\frac{R_{eq}}{R_L}$$

$$\rightarrow R_{eq} = \frac{I_2}{I_1}\frac{\Delta V_1}{\Delta V_2} R_L$$

But $I_1 \Delta V_1 = I_2 \Delta V_2$
$\Delta V_2 N_1 = \Delta V_1 N_2$

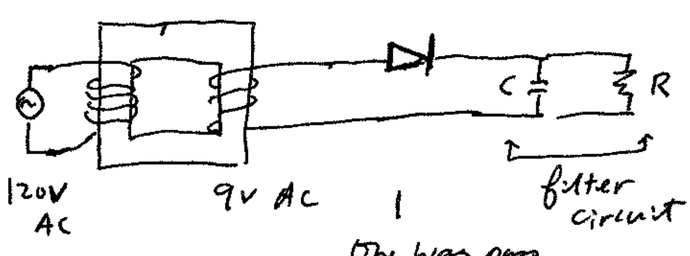
$$= \left(\frac{\Delta V_1}{\Delta V_2}\right)^2 R_L$$

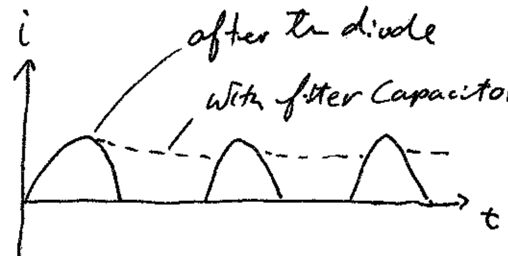
$$= \left(\frac{N_1}{N_2}\right)^2 R_L$$

33.9 Rectifiers and filters

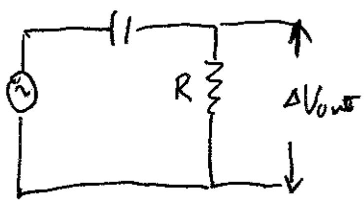
AC-DC Converter : 120v to 9 Volts
Alternating current to DC current.
→ rectification

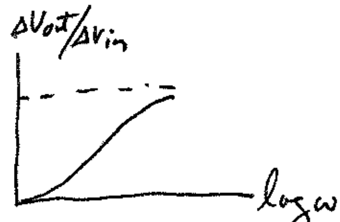
diode : A device element that conduct current in One direction



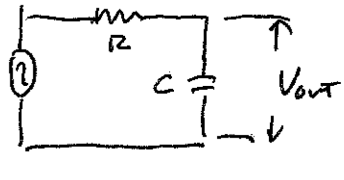


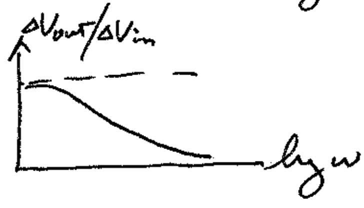
RC time Constant determines the time varying part of this AC current





high pass filter





low pass filter