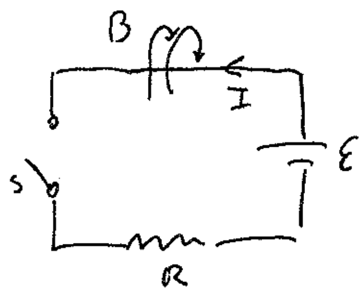
32.1 Self inductance

The current does not go from 0 to max, immediately



① when the switch is off, the current increases gradually, which cause an increase in the magnetic field

② the increased magnetic flux create an induced emf oppose the changing of the original magnetic flux.
→ also an gradualy increased current

③ The effect is called self-induction, created an self-induced emf.

$$\mathcal{E}_L \propto \frac{dI}{dt}$$

$$\mathcal{E}_L = -L\frac{dI}{dt}$$ → the self-induced emf is proportional to the time rate change of the current

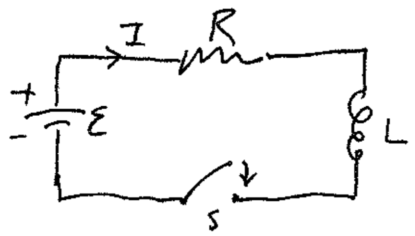
$L \equiv$ inductance $\equiv$ (coil symbol)

But from Faraday's law $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$

$\therefore L = \frac{N\Phi_B}{I}$ or $L = -\frac{\mathcal{E}_L}{dI/dt}$ [Henry] $= 1\frac{V\cdot s}{A}$

32.2 RL circuit

An inductor in a circuit oppose changes in the current in that circuit.



Using Kirchhoff's law

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

let $x = \frac{\mathcal{E}}{R} - I$ → $dx = -dI$

$$\therefore \quad x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\frac{dx}{x} = -\frac{R}{L}dt$$

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

$$\frac{dx}{x} = -\frac{R}{L}\,dt$$

$$\int_{x_0}^{x}\frac{dx}{x} = -\frac{R}{L}\int d\tau \quad\rightarrow\quad \ln\frac{x}{x_0} = -\frac{R}{L}\tau$$

$$\rightarrow x = x_0\, e^{-\frac{R}{L}t}$$

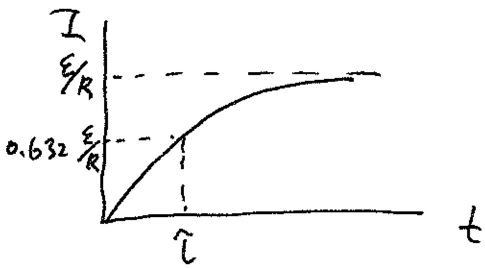
$I=0$ at $t=0$

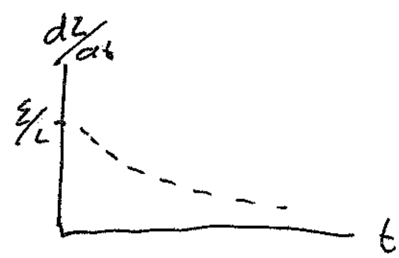
$x_0 = \frac{\varepsilon}{R}$

$$\therefore \frac{\varepsilon}{R} - I = \frac{\varepsilon}{R}e^{-\frac{R}{L}t}$$

$$\rightarrow I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

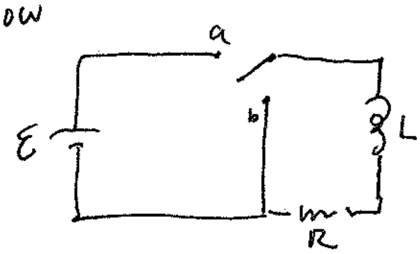
or $I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$, $\frac{dI(t)}{dt} = \frac{\varepsilon}{L}e^{-t/\tau}$





Switch open $t<0$
switch closed $t>0$

Now

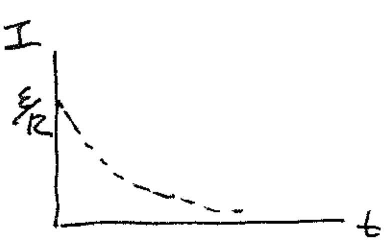


If the switch is thrown to b

$\rightarrow \varepsilon = 0$

$\rightarrow IR + L\frac{dI}{dt} = 0$

then $I(t) = \frac{\varepsilon}{R}e^{-t/\tau} = I_0\, e^{-t/\tau}$



32.3 Energy in a Magnetic field

— Because the emf induced in an inductor prevents a battery from establishing an instantaneously current, the battery must provide more energy than a circuit without an inductor

(32.6) $\mathcal{E} - IR - L\frac{dI}{dt} = 0$

$$\underbrace{I\mathcal{E}}_{\text{energy supplied by the battery}} = \underbrace{I^2R}_{\text{rate energy is delivered to the resistor}} - \underbrace{LI\frac{dI}{dt}}_{\text{energy rate stored in the inductor}}$$

$\therefore \frac{dU}{dt} = LI\frac{dI}{dt}$. $U \equiv$ energy stored in an inductor

$$\rightarrow U = \int dU = \int_0^I LI\,dI = L\int_0^I I\,dI = \frac{1}{2}LI^2$$

$$\therefore \underline{\underline{U = \frac{1}{2}LI^2}} \quad \text{energy stored in an inductor}$$

Note $U = \frac{1}{2}C(\Delta V)^2$ — energy stored in a Capacitor

Consider a Solenoid of N turns

$L = \mu_0 n^2 A\ell$ (32.5) example 32.1

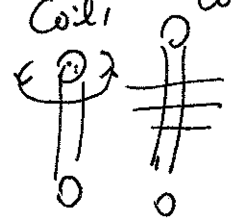
$B = \mu_0 n I$ (30.7)

$$\therefore U = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 A\ell \cdot \left(\frac{B}{\mu_0 n}\right)^2 = \frac{B^2}{2\mu_0}\underline{A\ell}$$

$u_B = \frac{U}{V} = \frac{1}{2}\frac{B^2}{\mu_0}$ in an inductor

$u_E = \frac{1}{2}\epsilon_0 E^2$ in a Capacitor

32.4 Mutual Inductance



The nearby circuits create magnetic flux passing each other

→ ~~Self~~ Mutual Induction

→ Mutual Inductance two adjacent Coils

$$M_{12} \equiv N_2 \frac{\Phi_{12}}{I_1}$$ Coil 2 with respect to Coil 1 due to current I_1 at Coil 2.

If I_1 varies with time

the emf induced by Coil 1 in Coil 2

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} I_1}{N_2}\right) = -M_{12}\frac{dI_1}{dt}$$

Similarly

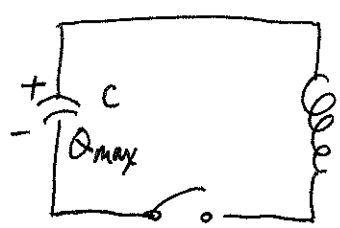
$$\mathcal{E}_1 = -M_{21}\frac{dI_2}{dt}$$

But $M_{12} = M_{21}$

$$\therefore\ \mathcal{E}_2 = -M\frac{dI_1}{dt}$$

$$\mathcal{E}_1 = -M\frac{dI_2}{dt}$$

32.5 Oscillations in an LC circuit



$$U = U_C + U_L = \frac{1}{2C}Q^2 + \frac{1}{2}LI^2$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Q^2}{2C} + \frac{LI^2}{2}\right)$$

$$= \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt}$$

the total energy of this system is constant

$$\frac{dU}{dt} = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C}\frac{dQ}{dt} + \frac{dQ}{dt}L\frac{d^2Q}{dt^2} = 0$$

$$\rightarrow \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\boxed{\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q} \rightarrow \text{Compare } \underbrace{\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x}_{\text{in a mechanical system}}$$

$$\rightarrow Q = Q_{max}\cos(\omega t + \phi), \quad \omega = \frac{1}{\sqrt{LC}}$$

$$I = \frac{dQ}{dt} = -\omega Q_{max}\sin(\omega t + \phi)$$

$I = 0$ @ $t = 0$

$Q = Q_{max}$ @ $t = 0$

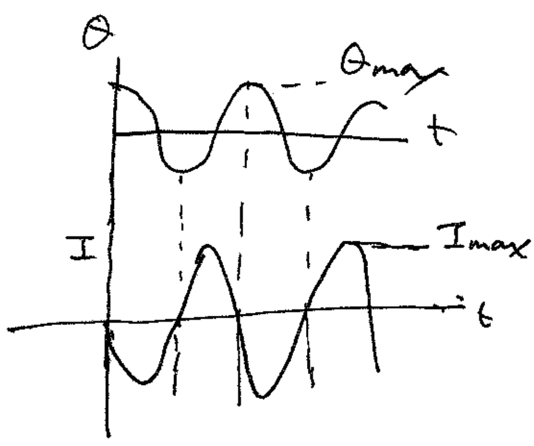
$$\therefore 0 = -\omega Q_{max}\sin\phi \rightarrow \phi = 0$$

$$\therefore Q(t) = Q_{max}\cos\omega t$$

$$I(t) = -\omega Q_{max}\sin\omega t = -I_{max}\sin\omega t$$

therefore

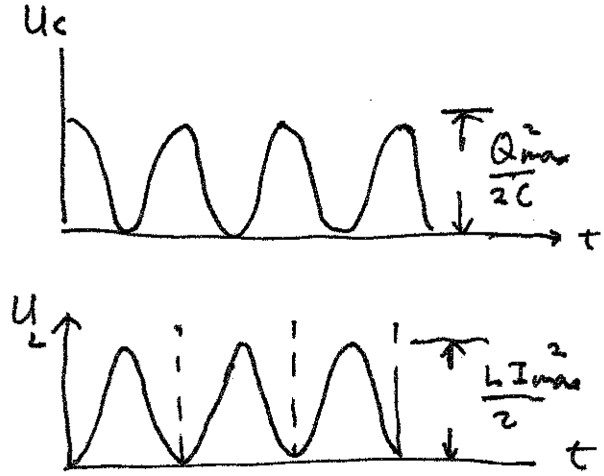
$$U = U_C + U_L = \frac{Q_{max}^2}{2C}\cos^2\omega t + \frac{1}{2}LI_{max}^2\sin^2\omega t$$



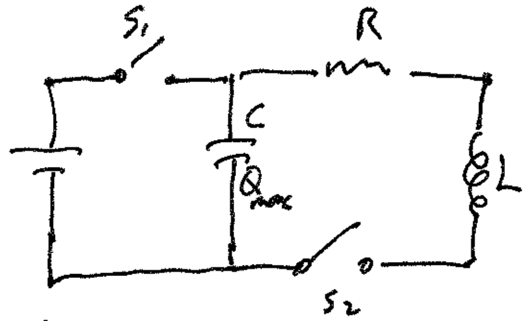
But $\frac{Q_{max}^2}{2C} = \frac{LI_{max}^2}{2}$

$$\therefore U = \frac{Q_{max}^2}{2C}(\cos^2\omega t + \sin^2\omega t)$$

$$= \frac{Q_{max}^2}{2C}$$



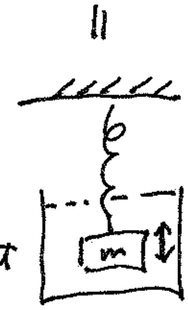
32-6 The RLC circuit



① S_1 closed. S_2 open
→ C charged to have Q_{max}

② S_1 open, S_2 closed
→ A ~~[illegible]~~ current establish
The total energy in C and L is

$$U = U_C + U_L = \frac{Q^2}{2C} + \frac{1}{2}LI^2$$



But the total energy is Not constant since the Resistor exists

$$-\frac{du}{dt} = -I^2R$$ → rate of energy transformation in a resistor

$$\therefore \frac{dU_{total}}{dt} \rightarrow LI\frac{dI}{dt} + \frac{Q}{C}\frac{dQ}{dt} = -I^2R$$

$$LI\frac{d^2Q}{dt^2} + I^2R + \frac{Q}{C}I = 0$$

$$L\frac{d^2Q}{dt^2} + IR + \frac{Q}{C} = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

Similar to $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$ (15.31)

— damped Oscillator

when R is small → light damped Oscillator

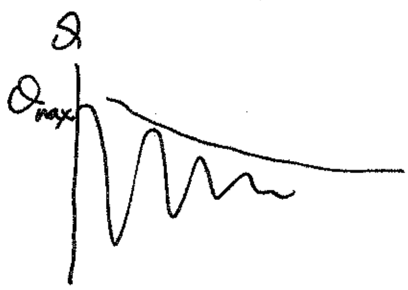
$$Q = Q_{max}\, e^{-\frac{Rt}{2L}} \cos \omega_d t$$

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{\frac{1}{2}}$$

① when $R \ll \sqrt{\frac{4L}{C}}$ $\omega_d \approx \omega$ for undamped $= \left[\frac{1}{LC}\right]^{\frac{1}{2}}$

② when $R_C = \sqrt{\frac{4L}{C}}$ critical damped

③ when $R > R_C$ overdamped



Check P.1021
P1022

Check Table 32.1