Chap 13 Universal gravitation

13.1 Newton's law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

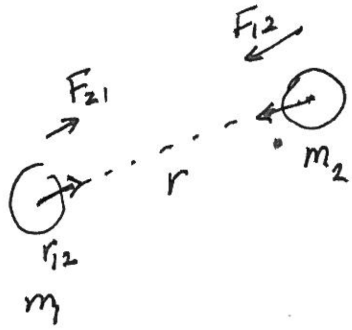
— Newton. 1687 "Principia"

← Newton's law of universal gravitation.

$$F_g = G\frac{m_1 m_2}{r^2} \qquad G = 6.673\times10^{-11}\ N\cdot m^2/Kg^2$$

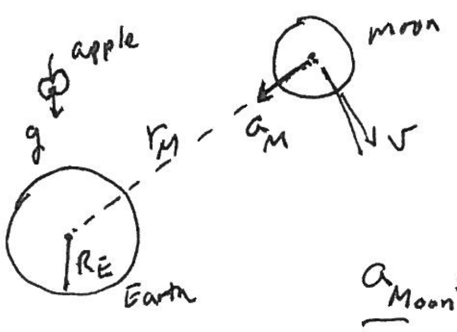
$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

$$= -\vec{F}_{21}$$



$$F_g = G\frac{M_E m}{R_E^2}$$: the magnitude of force exerted by earth on a particle of mass m near the earth surface

→ The gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass were concentrated at the center



$$\frac{a_{Moon}}{g} = \frac{(1/r_M)^2}{(1/R_E)^2} = \left(\frac{R_E}{R_M}\right)^2 = \left(\frac{6.37\times10^6}{3.84\times10^8}\right)^2 = 2.75\times10^{-4}$$

$$a_M = 2.75\times10^{-4}\times9.8 = 2.70\times10^{-3}\ m/sec^2$$

also

$$a_m = \frac{v^2}{r_M} = \frac{(2\pi r_M/T)^2}{r_M} = \frac{4\pi^2 r_M}{T^2} = \frac{4\pi^2(3.84\times10^8\,m)}{(2.36\times10^6\,sec)^2}$$

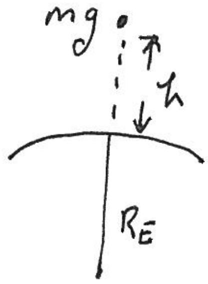
T = period of the moon
$= 27.32$ days
$= 2.36\times10^6$ sec

$$= 2.70\times10^{-3}\,m/sec^2$$

→ this provides a strong evidence that the inverse -square proportional nature of the gravitational force law

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$ – on earth surface



$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E+h)^2}$$

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E+h)^2}$$

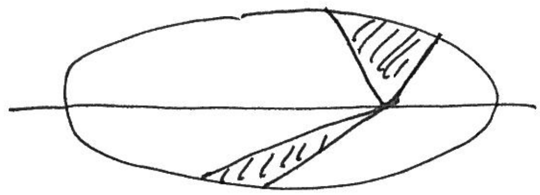
– g decreases with increase altitude

– when $r\to\infty$ $g\to0$

13.4 Kepler's law and the motion of planets

Kepler's law

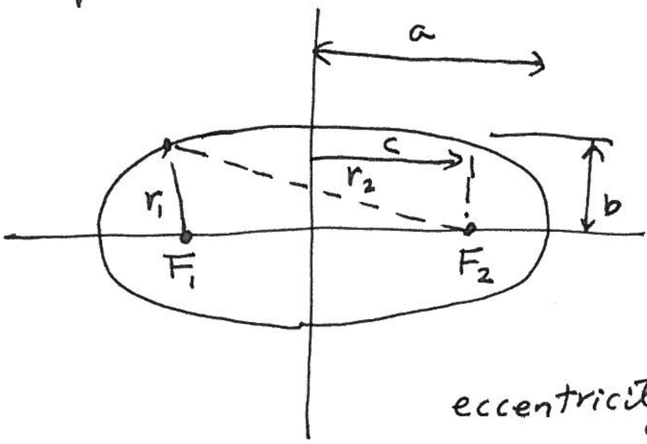
1) All planets move in elliptical orbits with the sun at one focus – Elliptical law
2) The radius vector drawn from the sun to a planet sweeps out equal areas in equal time intervals



Same area in a same period T

3) The square of the ~~orbi~~ orbital period of any planet is proportional to the cube of the semimajor axis of the elliptical orbit

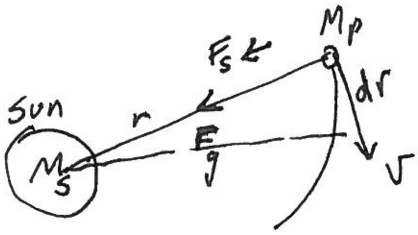
1) Kepler's first law



eccentricity $e = \frac{c}{a}$

describe the general shape of the ellipse.

2) 2^{nd} law – Consequence of angular momentum conservation



The torque due to the force on the planet is zero (in the direction of the planet, there is no force)

$\tau = \vec{r} \times \vec{F} = r \times F(r)\hat{r} = 0$

$\tau = \frac{dL}{dt} = 0 \quad L = \text{constant}$

$L = r \times p = M_p \vec{r} \times \vec{v} = \text{constant}$

$$L = r \times p = M_p \vec{r} \times \vec{v} = \text{Constant}$$

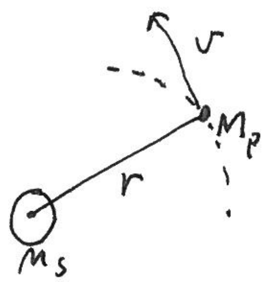
$$dA = \frac{1}{2} |\vec{r} \times d\vec{r}| = \frac{1}{2} |r \times \vec{v} dt| = \frac{L}{2M_p} dt$$

or ds

$$\frac{dA}{dt} = \frac{L}{2M_p} = \text{constant}, \text{ regardless the orbit}$$

— The radius vector from the sun to any planet sweeps out equal areas in equal time.

3) Kepler's 3rd law



$$\frac{G M_s M_p}{r^2} = \frac{M_p v^2}{r}$$

$$= \frac{M_p (2\pi r/T)^2}{r}$$

$$T = \frac{2\pi r}{v}$$

$$v = 2\pi r/T$$

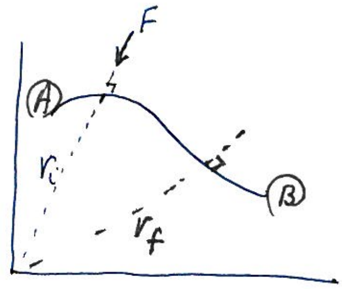
$$T^2 = \left(\frac{4\pi^2}{G M_s}\right) r^3$$

$$\frac{T^2}{r^3} = \left(\frac{4\pi^2}{G M_s}\right) = \text{Constant}$$

$$= K_s, \quad K_s \equiv \frac{4\pi^2}{G M_s}$$

Check Table 13.2, P. 399

13.6 Gravitational potential



for a central force, since the force is perpendicular to the arc. therefore the work done for a central force is independent of the path

$$dW = F \cdot dr = F(r)\,dr$$

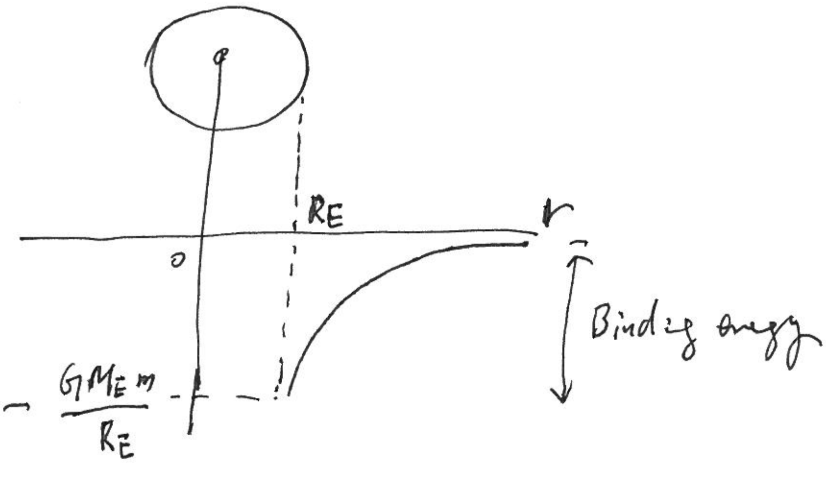
$$W = \int dW = \int_{r_i}^{r_f} F(r)\,dr$$

$$\Delta U = -\Delta W = U_f - U_i = -\int_{r_i}^{r_f} F(r)\,dr$$

$$\therefore\ U_f - U_i = -\int_{r_i}^{r_f} -\frac{GM_E m}{r^2}\,dr = GM_E m \int_{r_i}^{r_f} \frac{1}{r^2}\,dr$$

$$= -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$\therefore\ U(r) = -\frac{GM_1 m_2}{r}$$



from the energy point of view

$$E = K + U$$

$$= \frac{1}{2}mv^2 - \frac{GMm}{r}$$

But $\frac{GMm}{r^2} = ma = \frac{mv^2}{r}$

$$\therefore \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Therefore $E = \frac{GMm}{2r} - \frac{GMm}{r}$

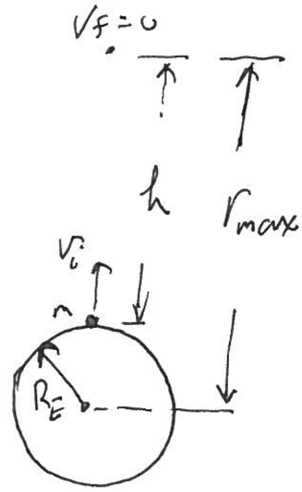
$$E = -\frac{GMm}{2r}$$ → Circular Orbits

→ total energy of the system

$$= -\frac{GMm}{2a}$$ – total energy is negative.

need energy to break this system,

⇒ Both the total energy and total angular momentum are constant.



$$\frac{1}{2}mV_i^2 - \frac{GM_Em}{R_E} = -\frac{GM_Em}{r_{max}}$$

if $V_f = 0$

$$\rightarrow V_i^2 = 2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)$$

Max. height $= r_{max} = R_E + h$

if $r_{max} \to \infty$, that is this object will be escape from earth

$$V_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

for a rocket $V_{esc} = \sqrt{\frac{2GM_E}{R_E}} = 1.12 \times 10^4$ m/sec

~ 10 km/sec