11.1 Vectorproduct

Torque $= rF\sin\phi$ = a vector

$\equiv \vec{r} \times \vec{F}$

cross product

= Vector product

$\vec{C} = \vec{A} \times \vec{B}$

$\equiv AB\sin\theta$, with direction perpendicular to the plan of $AB\sin\theta$

1) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

2) If $\vec{A} \parallel \vec{B}$. $\rightarrow$ $\vec{A} \times \vec{B} = 0$

3) If $\vec{A} \perp B$ $\rightarrow$ $\vec{A} \times \vec{B} = AB\sin 90° = AB$

4) distribution law : $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

5) $\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + A \times \frac{d\vec{B}}{dt}$

6) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$

$\hat{j} \times k = -\hat{k} \times \hat{j} = i$

$k \times i = -i \times k = j$

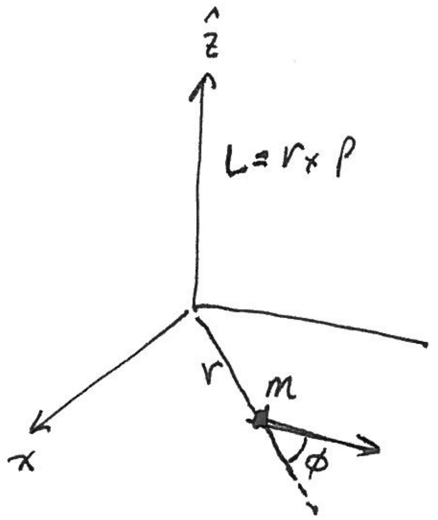
7) $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$

$= (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$

11.2. Angular Momentum

$$\sum \tau = \vec{r} \times \sum F = r \times \frac{dP}{dt}$$

$$= r \times \frac{dp}{dt} + \underbrace{\frac{dr}{dt} \times p}_{=0}$$



$$\sum \tau = \frac{d}{dt}(\vec{r} \times \vec{p}) \rightarrow \text{Angular}$$

$$\sum F = \frac{d}{dt}(\vec{p}) \rightarrow \text{linear translation}$$

∴ Define Angular Momentum $\underbrace{\vec{L} \equiv \vec{r} \times \vec{p}}$ $= mvr\sin\phi$

$$\sum \tau = \frac{d}{dt}(\vec{L})$$

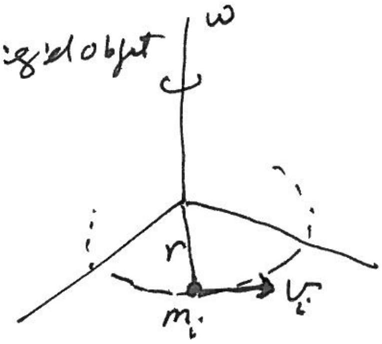
— torque is equal to the rate of change of the angular momentum

for a system of particles

$$\sum F_{ext} = \frac{dP_{tot}}{dt}$$

$$\sum_i \tau_i = \sum_i \frac{dL_i}{dt} = \frac{d}{dt}\vec{L}_{tot}$$

10.3 Angular momentum of a rotating rigid object



$$L_i = m_i v_i r_i = m_i r_i^2 \omega$$

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega$$

$$= \left(\sum_i m_i r_i^2\right)\omega$$

$$= I\omega$$

$$\frac{dL_z}{dt} = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha = \sum \tau_{ext} = I\alpha$$

11.4 Conservation of Angular Momentum

In an isolated system, the total angular moment of a system is constant in both magnitude and direction, if there is no external torque.

$$\sum \tau_{ext} = \frac{d}{dt} L_{tot} = 0$$

or $L_{total} = $ Constant or $L_i = L_f$

or $I_i \omega_i = I_f \omega_f$

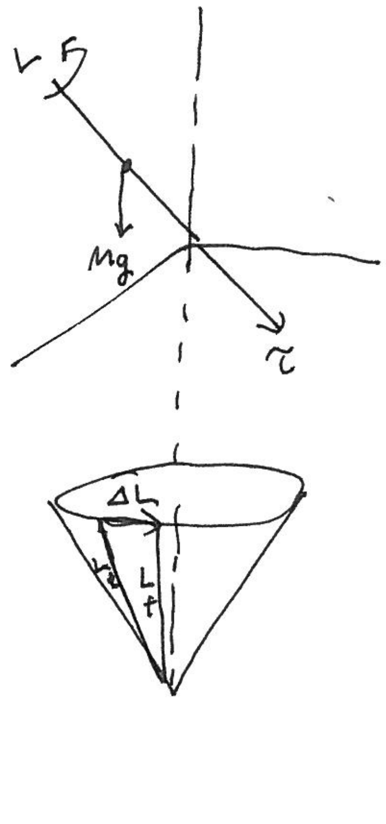
$\therefore$ for an isolated system
$E_i = E_f$
$P_i = P_f$
$L_i = L_f$

11.5 The Motion of Gyroscopes and Tops

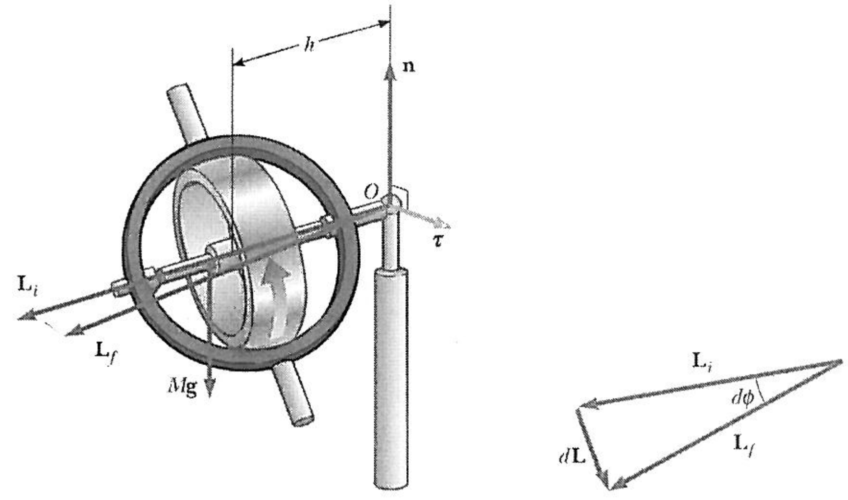
Presessional motion of a top (Fig 11.14) page 350

$$\tau = \frac{dL}{dt}$$

$\therefore \tau \parallel \Delta L$



P11-4



$\tau = \frac{dL}{dt}$

(a) (b)

Mg produces a torque τ

$\tau \parallel \Delta L$

$\Delta L \parallel xz$ plane

$\Delta L \perp L$ if $\Delta t \to 0$

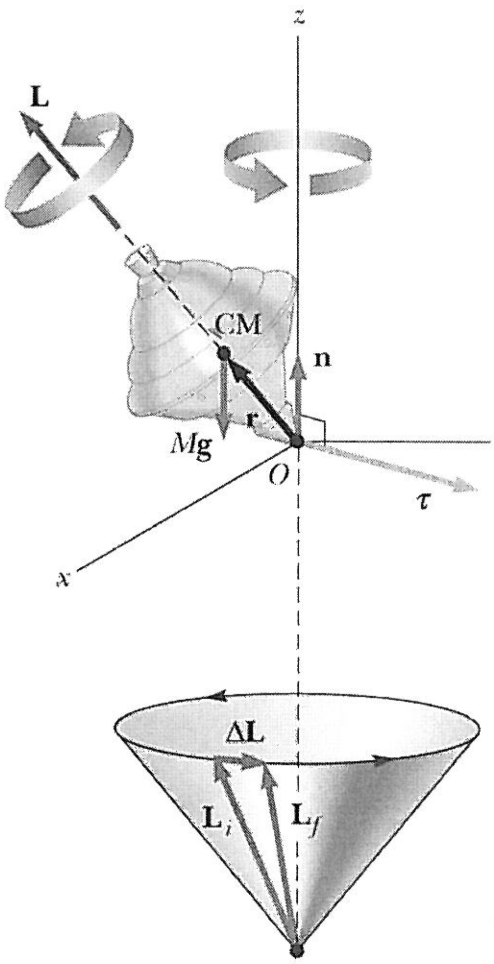
$$\sin(d\phi) \approx d\phi = \frac{dL}{L} = \frac{\tau\, dt}{L}$$

$$= \frac{Mgh\, dt}{L}$$

$L = I\omega$

$$\therefore \; d\phi = \frac{mgh\, dt}{I\omega}$$

$$\boxed{\frac{d\phi}{dt} = \omega_p = \frac{mgh}{I\omega}}$$



(a)

(b)