



SN: \_\_\_\_\_, Name: \_\_\_\_\_

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables **are not** allowed. Please hand in both solution and this problem sheet. **ABSOLUTELY NO CHEATING!** This is a one-hour examine

**Problems (10% for each sub-problem, total 100%)**

- 1. Charged Spheres: (60%)** (a) A solid spherical **conductor**, radius  $R$ , is charged to a total charge  $Q$ . (a1) How will the charges distributed in the sphere and why? (a2) What is the electric potential and electric field due to the sphere as a function of distance  $r$  from the center of the sphere? (a3) plot the schematic of the electric and potential at various points as a function of distance  $r$  from the center. (b) A solid spherical **insulator**, radius  $R$ , is charged to a total charge  $Q$ . (b1) How will the charges distributed in the sphere and why? (b2, b3) What is the electric field and electric potential of a point inside this sphere as a function of distance  $r$  from the center of the sphere ( $r < R$ )?
- 2. Electric Dipole (20%)** An electric dipole ( $P$ , charge separation  $d$ ) is placed in a uniform electric field  $E$ , making an angle  $\theta$  with the electric field. (a) What is the torque of the dipole feels due to the electric force? (b) If the dipole is making a  $90^\circ$  angle with the electric field, what is the work (energy) needed to flip the dipole to make angle  $\theta$  with the electric field?
- 3. Entropy (20%)** The entropy changes in an adiabatic free expansion. (a) Explain what does it mean by "an adiabatic free expansion"? (b) What are the entropy changes in an adiabatic free expansion? In both of the problems, you need to assume the needed parameters

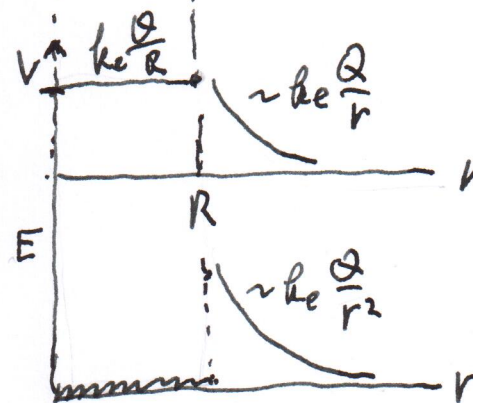
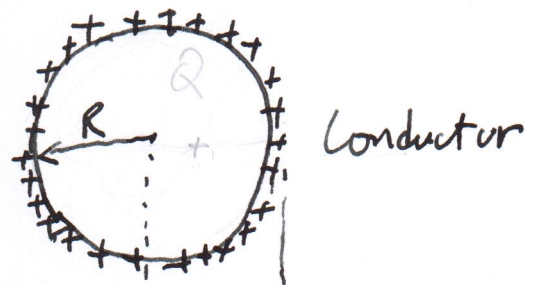
1. (a) For a spherical solid conductor.  
Since the charges can move freely.

(1) the charges will evenly distributed on the surface of the sphere, because the charges will repel one another

(2) The surface potential is a constant

$$V_{\text{surface}} = k_e \frac{Q}{R}$$

There is no electric field inside the sphere, since there is no charge inside



(b) For an insulator, the charges can not move, the charge will evenly distributed in the sphere when  $r < R$  inside the sphere

$$E_r(r) = k_e \frac{Q_{\text{in}}}{r^2}$$

$$= k_e \frac{1}{r^2} \cdot \frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3$$

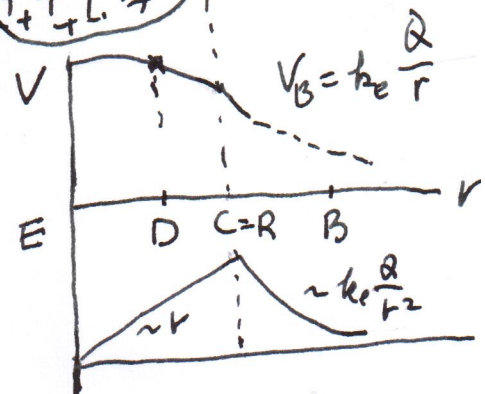
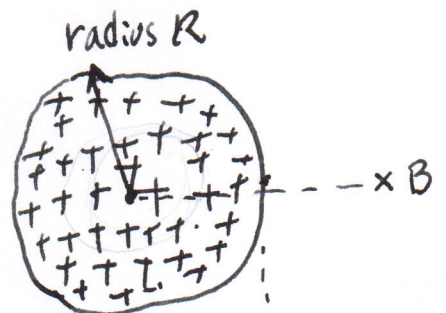
$$= k_e \frac{Q}{R^3} \cdot r$$

$$V_D = \frac{k_e Q}{2R} \left( 3 - \frac{r^2}{R^2} \right)$$

$$V_D - V_C = - \int_R^r E_r dr = - \int_R^r k_e \frac{Q}{R^3} r dr$$

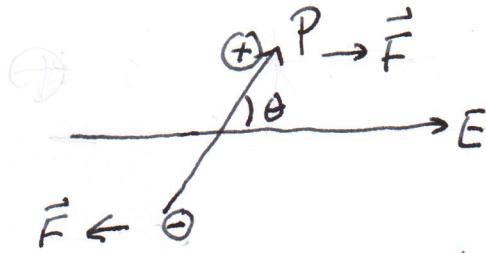
$$= \dots = k_e \frac{Q}{2R^3} (R^2 - r^2) \text{ but } V_C = k_e \frac{Q}{R}$$

$$\therefore V_D = V_C + k_e \frac{Q}{2R^3} (R^2 - r^2) = \frac{k_e Q}{2R} \left( 3 - \frac{r^2}{R^2} \right)$$





2. (a)



The dipole will be subject to two forces on its each end of the charges.

$$\tau = F \frac{d}{2} \sin \theta + F \frac{d}{2} \sin \theta = F d \sin \theta$$

$$= q E d \sin \theta$$

$$= P E \sin \theta = \vec{P} \times \vec{E}$$

$$= -P E \sin \theta \text{ (negative torque)}$$

negative sign comes from the rotation direction. This will give rise to the rotation motion of the dipole.

(b)

The potential energy of the dipole is

$$U = -W = - \int_{90^\circ}^{\theta} \tau d\theta$$

$$= - \int_{90^\circ}^{\theta} -P E \sin \theta$$

$$= \int_{90^\circ}^{\theta} P E \sin \theta$$

$$= -P E \cos \theta$$

$$= -\vec{P} \cdot \vec{E}$$

3 (a) adiabatic means at the same temperature, or the temperature is not changing.

free expansion means:  $V_i \rightarrow V_f$

$W = \text{work}$

$$dW = 0$$

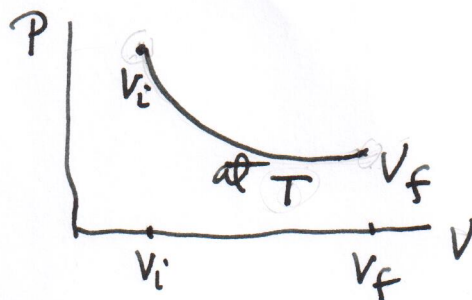
$Q = \text{heat}$

$$dQ = 0$$

$E_{\text{int}} = \text{internal energy}$

$$dE_{\text{int}} = 0$$

(b) For free expansion like this ~~one~~ one, we can find a reversible process that has the same initial and final states  $\rightarrow$  i.e. an isothermal reversible expansion



Then

$$\Delta S = \int_{V_i}^{V_f} \frac{dQ_r}{T}$$

$$\text{but } dE_{\text{int}} = 0 \Rightarrow dQ_r = dW$$

$$= \frac{1}{T} \int dW = nR \ln \left( \frac{V_f}{V_i} \right)$$

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you don't need to carry out this,