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General Physics I, Final 1
PHYS1000AA, AB, AC, Class year 112-1
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SN:__________________, Name:__________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are not allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - .....$. Time average $\overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T}\int_0^T x(t)^n dt$

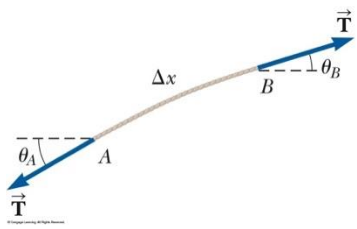
For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$ the general solution of this equation is

$x(t) = x_0 \cos(at + \phi)$, where x_0 is the maximum, and ϕ is the phase angle.

$N_A=6\times10^{23}$, R = Gas Constant= 8.31 J/mole K, room temperature =300K, 1atm=1.01×105 Pa.

Problems (5 Problems, total 125%, 25% each)

1. **Escape velocity:** Calculate the escape velocity of a body starting from the surface of the earth. Ignore air friction. The mass of the earth is ***Me***, and its radius is ***Re***.
2. **First law of thermodynamics:** What is the work done by an ideal gas in expanding adiabatically from a state (P_1, V_1) to (P_2, V_2)?
3. **Gravitational Force:** Let the mass of a planet be M, radius R. The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. (a) Why? (b) Show that the corresponding shortest period of rotation is $T = \sqrt{\frac{3\pi}{G\rho}}$. Where ρ is the uniform density of the spherical planet?
4. **Simple harmonic Oscillation:** A spring mass oscillator has a total energy E_0 and an amplitude of x_0. Let let spring constant is k. (a) How large will the kinetic energy (E_k) and potential energy (E_p) be for it when x=x_0/2? (b) For what value of x will $E_k = E_p$?
5. **Wave Equation**: Use the figure to the right to derive the wave equation for a string. If you focused on this section of the string, you can find the mass of the string is oscillating vertically (y-direction) that is it is perpendicular to the wave's travelling direction (say, to the right or in the +x direction). Let the same section, suppose the vibration of the string can be represented as a function $y(x, t)$; a function of both x and t. Prove (or derive) that the wave equation describing this wave motion is $\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}$, where μ is the mass density and T is the tension in the string.



1. Escape Velocity

The potential energy change of this body is ΔP_E

$$\Delta P_E = \frac{GM_e m}{R_e} - \frac{GM_e m}{r}$$ r= position of body

To escape, $r \to \infty$

then

$$\Delta P_E = \frac{GM_e m}{R_e}$$

when escape. $V = 0$

Energy conservation is $\Delta K + \Delta P_E = 0$

$$(K_f - K_i) + \frac{GM_e m}{R_e} - \frac{GM_e m}{r_\infty} = 0$$

$$\therefore \left(0 - \frac{1}{2} m V_e\right) + \frac{GM_e m}{R_e} - 0 = 0$$

$\rightarrow V_e = \sqrt{\frac{2GM_e}{R_e}}$ $\Rightarrow$ if you plug in numbers it is, $V_e = 11.2 \text{ km/sec}$

2. For an ideal gas going adiabatically expansion

$PV^{\gamma} = \text{constant}. \quad \gamma \equiv \frac{C_p}{C_v}$

let the constant be C, then

$$PV^{\gamma} = C, \text{ or } P = CV^{-\gamma}$$

The work done is $W = \int_{V_1}^{V_2} P\,dV = C\int_{V_1}^{V_2} V^{-\gamma}\,dV$

$$W = C\left[\frac{1}{-\gamma+1} V^{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{C \cdot 1}{1-\gamma}\left[V_1^{1-\gamma} - V_2^{1-\gamma}\right]$$

$$= \frac{C\left(V_1^{1-\gamma} - V_2^{1-\gamma}\right)}{\gamma - 1}$$

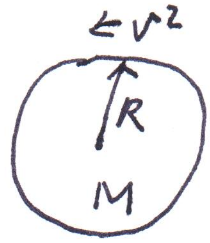
But $PV^{\gamma} = C \Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma} = C$

$$\therefore W = \frac{P_1V_1^{\gamma}V_1^{1-\gamma} - P_2V_2^{\gamma}V_2^{1-\gamma}}{\gamma - 1}$$

$$= \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

3 The magnitude of the gravitational force exerted by the planet on an object of mass m at the surface is

$$F = \frac{GMm}{R^2} = \frac{mv^2}{R}$$



Let M = mass of the planet
m = mass of the object
R = radius of the planet.

(a) If the gravitational force is less than the centrifugal force, then the object will fly off.

(b) $$M = \frac{4}{3}\pi R^3 \rho$$

$\rho \equiv$ the density of the planet

$$v = \frac{2\pi R}{T}$$

$T \equiv$ the period of the revolution

Plug in v into the equation

$$\frac{G \cdot \frac{4}{3}\pi R^3 \rho \cdot m}{R^2} = \frac{m\left(\frac{2\pi R}{T}\right)^2}{R}$$

$$\rightarrow \quad T = \sqrt{\frac{3\pi}{G\rho}}$$

4. The total energy is $E_0 = E_k + E_p$

or, $\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + E_k$, let the spring constant be k

(a) When $x = \frac{1}{2}x_0$, $E_p = \frac{1}{2}k\left(\frac{1}{2}x_0\right)^2$

$= \frac{1}{4}E_0$

$E_k = E_0 - E_p$

$= \frac{1}{2}kx_0^2 - \frac{1}{4}E_0$

$= E_0 - \frac{1}{4}E_0$

$= \frac{3}{4}E_0$

(b) If $E_p = E_k$ @ $E_p = \frac{1}{2}E_0$

$= \frac{1}{2}\,\frac{1}{2}kx_0^2$

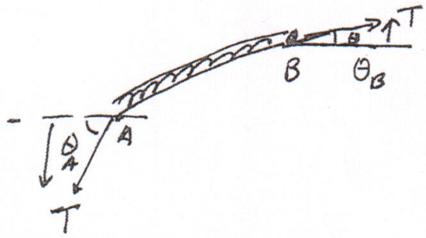
$= \frac{1}{4}kx_0^2$

$= \frac{1}{2}kx^2$

$\therefore x = \frac{x_0}{\sqrt{2}}$

5.

Net force in $\hat{y}$ direction



$$\sum F_y = T\sin\theta_B - T\sin\theta_A$$

$$= T(\sin\theta_B - \sin\theta_A) \quad \text{if } \theta \text{ is small}, \ \sin\theta \approx \tan\theta$$

$$\approx T(\tan\theta_B - \tan\theta_A)$$

$$= T\left(\frac{\partial y}{\partial x}\Big|_{\text{at } B} - \frac{\partial y}{\partial x}\Big|_A\right) = ma_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\therefore \ \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\text{or} \quad \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - \left(\frac{\partial y}{\partial x}\right)_A}{\Delta x}$$

$$= \frac{\partial^2 y}{\partial x^2} \qquad \text{Note: } \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\therefore \ \boxed{\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}} \quad \text{or} \quad \boxed{\frac{\mu}{T}\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}}$$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(kx - \omega t)$$

$$\frac{-\omega^2\mu}{T}\sin(kx - \omega t) = -k^2\sin(kx - \omega t)$$

$$\rightarrow k^2 = \frac{\mu}{T}\omega^2, \quad v = \frac{\omega}{k}$$

$$\therefore \ v^2 = \frac{\omega^2}{k^2} = \frac{T}{\mu} \quad \rightarrow \boxed{v = \sqrt{\frac{T}{\mu}}}$$