Chapter 29, Magnetic Field

1. Charge produces a vector field → electric field $\vec{E}$
   magnet produces a vector field → Magnetic field $\vec{B}$,
   1) magnets on the refrigerator door.
   2) electromagnet; current in a coil wound around an iron core, → produced by moving current.
   3) permanent magnets,
      → Elementary particles that have an instrinsic magnetic field around them. When these all add up, make up the magnetic properties of permanent magnets.

2. Definition of $\vec{B}$, magnetic force

$$\vec{E} \equiv \frac{\vec{F}_E}{q}$$ q : Charge, may be single charge.

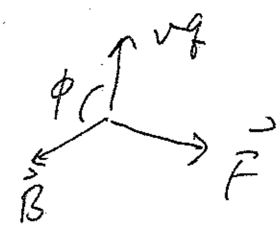
$$\vec{B} = \frac{\vec{F}_B}{q_B}$$ m_B : magnetic mono pole. But this is not existing yet.

→ we define in a different way.
this can be defined in terms of $\vec{F_B}$ exerted on a moving electrically charge test particles.

$$m_B \propto q v$$

$\therefore$ $F_B = q\vec{v} \times \vec{B}$ — experimental results

$\Rightarrow$ $B \equiv \frac{F_B}{|q|v}$

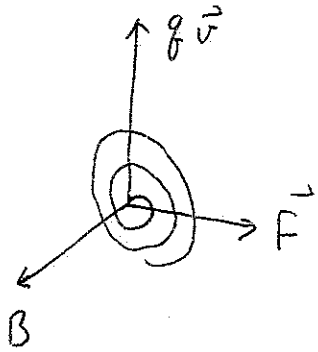


$F_B = |q| v B \sin\phi$

$F_B = 0$ if charged particle is stationary

$F_B \perp \vec{v}$, $\vec{F}$ will not change the ~~velocity~~ speed of the particle.

$\vec{F} = q\,\vec{v} \times \vec{B}$



$[\vec{F}]$ = Newton
$[q]$ = Coulomb
$[v]$ = m/sec

→ $[B] = \dfrac{\text{Newton} \cdot \text{sec}}{\text{Coulomb} \cdot \text{m}}$ = 1 tesla = 1 T

or $1\,T = 1 \dfrac{N}{C \cdot m/s}$

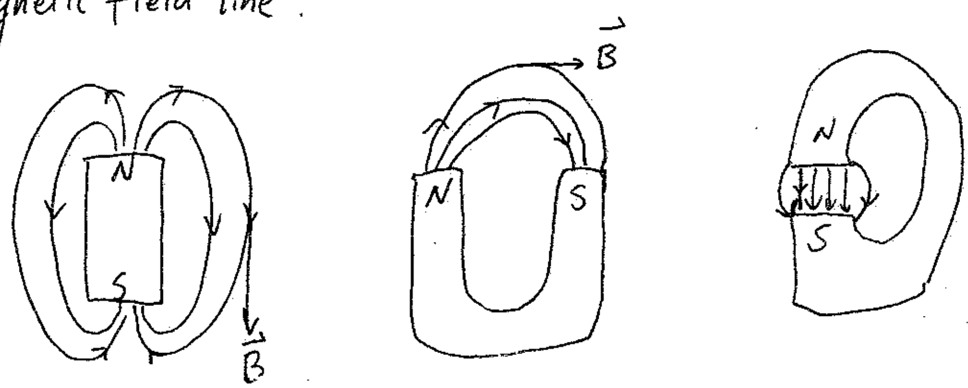
$\dfrac{C}{\text{sec}}$ = Ampere. $1\,T = 1 \dfrac{N}{A \cdot m}$

Check Table 29-1 ~~page~~ page 899

$10^{-4}\,T = 100\,\mu T = 1$ gauss

magnetic field at earth surface

3. Magnetic field line.



① The tangent of the line at any given pt represent the direction of the $\vec{B}$ field at that point

② The density of the field line represent the strength of the $\vec{B}$ field.

③ Field lines emerge from the North pole and end at the South pole.

④ Opposite magnetic poles attract each other, and like magnetic poles repel each other.

⑤ Compass with a small slend bar magnet can be used to detect the earth magnetic field.

S-P-29-1 $EK = 5.3\,MeV = \frac{1}{2}mv^2$

$$V = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times 5.3\times 10^{6}\times 1.6\times 10^{-19}\,J/eV}{1.67\times 10^{-27}\,kg}} \qquad M_p = 1.67\times 10^{-27}\,kg$$

$$= 3.2\times 10^{7}\,m/s$$

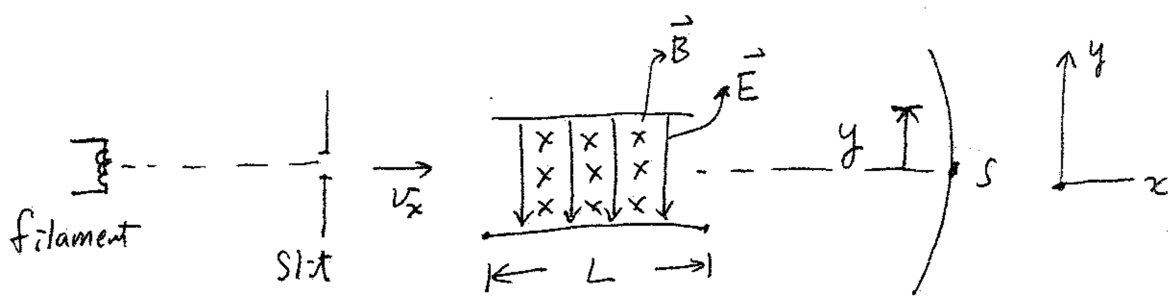
$$F_B = |q|\,vB\sin\phi = 1.6\times 10^{-19}\,C\cdot 3.2\times 10^{7}\times 1.2\times 10^{-3}\,T\cdot \sin 90^\circ$$

$$= 6.1\times 10^{-15}\,N$$

$$a = \frac{F_B}{m_p} = \frac{6.1\times 10^{-15}\,N}{1.67\times 10^{-27}\,kg} = 3.7\times 10^{12}\,m/s^2$$

* Small force, but acts on a small mass results in a huge acceleration.

4. Discovery of electrons by J.J. Thomson @ Cambridge Univ.



Modern version of J.J. Thomson's apparatus

① $\vec{E} = \vec{B} = 0$, Undeflected beam on S.
② $\vec{E}$ on, $\vec{B} = 0$, Beam is up by $\vec{E}$ field
③ $\vec{E}$ same, on, $\vec{B}$ increases until beam is steered back to S

$$a_y = \frac{F_E}{m} = \frac{qE}{m}, \quad y = \frac{1}{2}a_y t^2, \quad L = v_x t, \quad t = \frac{L}{v_x}$$

$$\therefore\ y_E = \frac{1}{2}\frac{qE}{m}\frac{L^2}{v_x^2}$$ → the direction of the deflection depends on the sign of the particle. in this set up. the particles are negatively charged.

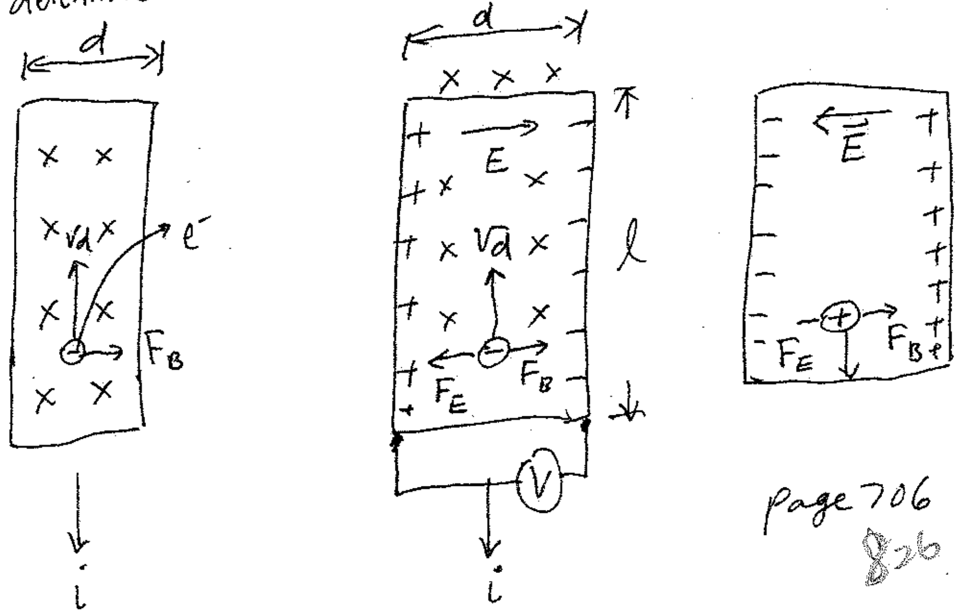
if $qE = qv_xB\sin 90^\circ = qv_xB$

$$v_x = \frac{E}{B}$$

$$\Rightarrow y_E = \frac{1}{2}\frac{qE}{m}\frac{L^2}{v_x^2} = \frac{1}{2}\frac{qE}{m}\frac{L^2B^2}{E^2} \Rightarrow \boxed{\frac{m}{q} = \frac{B^2L^2}{2yE}} \Rightarrow m_e$$

5 Hall Effect – Edwin H. Hall (1879) Johns Hopkins

— determine Charge carrier's sign, electrons move in the conductor.
— determine number density of the charge carrier in a conductor.



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① F_B makes electrons move to the right.
② The separation of electrons and positrons create an $\vec{E}$ field, cancel the F_B with F_E. There will be no further separation of the charges., V_d will then move upward.

Hall potential $V = Ed$, can be measured with a Voltmeter, deciding the potentia

③ if the carrier were positively charged, the Voltmeter will read diffently, this is not happening.

④ when $\vec{E}$ and $\vec{B}$ are in Balance

$$eE = e v_d B$$

$e = q$

But eq.27-7, $v_d = \frac{J}{ne} = \frac{i}{neA}$

$$V = \frac{iBd}{nqA} = \frac{iB}{nq} t$$

$$V = Ed$$

$R_H = \frac{1}{nq}$ = Hall coefficien

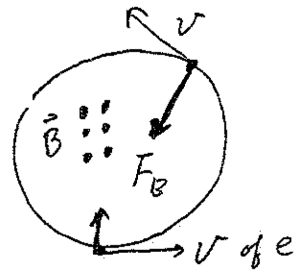
$\Rightarrow \quad E = v_d B \Rightarrow \quad \frac{V}{d} = \frac{iB}{neA}, \quad l = \frac{A}{d}$

$$\Rightarrow \boxed{n = \frac{Bi}{Vle}}$$ charge carrier's number density.

6. Circulating charged particles

i) $v \perp B$

$$F = ma = \frac{mv^2}{r}$$



$$qvB = \frac{mv^2}{r}$$

$$\boxed{r = \frac{mv}{qB}}$$

$$T = \text{period} = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

$$f = \text{frequency} = \frac{1}{T} = \frac{qB}{2\pi m}$$

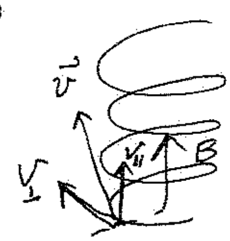
$$\omega = 2\pi f = \frac{qB}{m} = \text{angular frequency}$$

* T, f, ω doesn't depend on the speed of the charged particles. (iff $v \ll c$)

ii) Helical paths. – If v has component along $\vec{B}$, $\vec{v}_{\parallel}$

$V_{\parallel} = V\cos\phi$, $V_{\perp} = V\sin\phi$
(pitch) (radius)

$\vec{V}_{\parallel}$ will make the charge partical move along the direction of $\vec{B}$

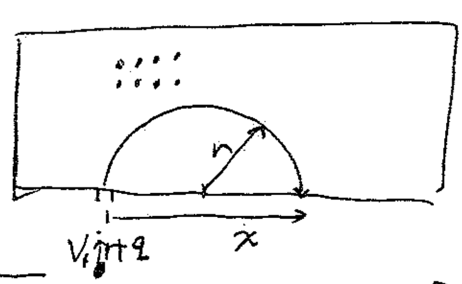


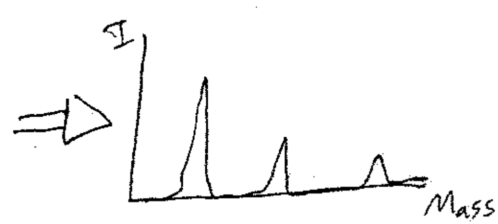
one method, use to measure ion

7. Mass spectrometer (sp-29-3)

$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}}$$





$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}}$$

$$x = 2r = \frac{2m}{qB}\sqrt{\frac{2qV}{m}}$$

$$qvB = \frac{mv^2}{r}$$

$$\boxed{m = \frac{B^2 q x^2}{8V}}$$

determining x, can decide m

粒子迴旋加速器

8 Cyclotrons and Synchrotrons

① cyclotron

i) |⟵ V potential difference ⟶, ~~formed by "D"~~

$\oplus \rightarrow$ $E_K = eV$, for high energy. V becomes difficult.

ii) a better way is to let the proton circulating in a circle. each time, give it a "kick",

for example (100 eV) × 100 revolution $= 10^4$ eV

electric D: Oscillator

$qvB = \frac{mv^2}{r}$

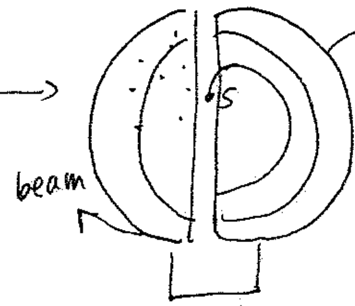
$v = \frac{qBr}{m}$

$K = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(\frac{qBr}{m}\right)^2$

$T = \frac{2\pi r}{v}$

$= \frac{2\pi r}{\frac{qBr}{m}}$



$f = f_{osc}$ $\quad f = T^{-1} = \frac{qB}{2\pi m} = f_{osc}$ ↑ circulating frequency

$qB = 2\pi m f_{osc}$ (~~28-19~~ 29-18)

$f = \frac{qB}{2\pi m}$

Oscillator, $B \approx 1.5T = 10^4$ Gauss

② Synchrotron.: when $E_K > 50$ MeV. (v is very large)

i) $qB = 2\pi m f$ is true only when $v \sim$ small, $v \ll c$

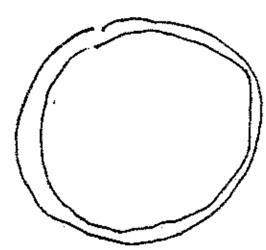
ii) For 500 GeV proton, $B = 1.5T$, $r = 1.1$ km

Area of magnet $\simeq 10^6 m^2$ → too expensive

⇒ (A) $\vec{B}$ and Oscillator are made to ~~varied~~ vary with time.

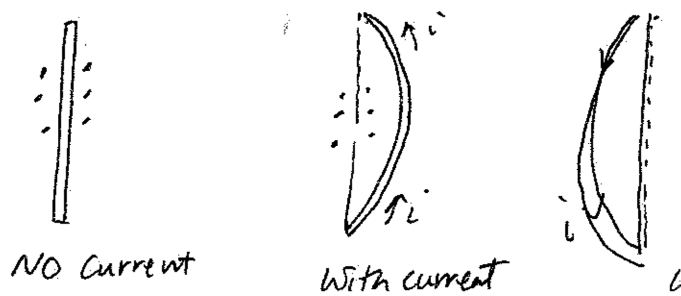
(B), the frequency of the proton remain in step with the oscillator at all time, f vary each oscillation

(C) protons follow a circular path instead spiral.



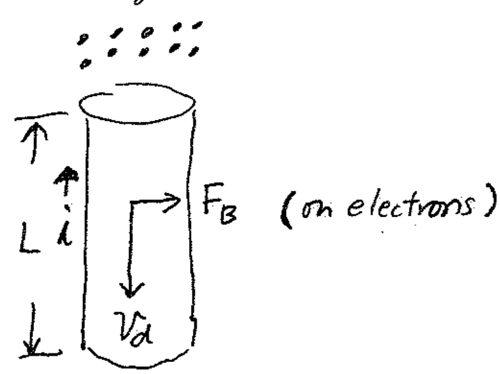
9. Magnetic Force on a current-carrying wire

→ Hall Effect tells us that Magnetic force push electrons side way in a conducting wire.
→ Electrons can not escape from the conductor,
→ Magnetic force transmitted to the wire.



$F_B = e V_d B$ in both cases. Only the direction is different.

Consider a wire of length L



$$q = i t = i \frac{L}{V_d}$$

$$\therefore F_B = q v_d B \sin\phi$$
$$= i \frac{L}{V_d} V_d B \sin 90°$$

$$\boxed{F_B = i L B} \Rightarrow \vec{F}_B = i \vec{L} \times \vec{B}$$

$\vec{L}$ has the direction of the current.

$d\vec{F}_B = i\, d\vec{L} \times \vec{B}$ for non straight wire.

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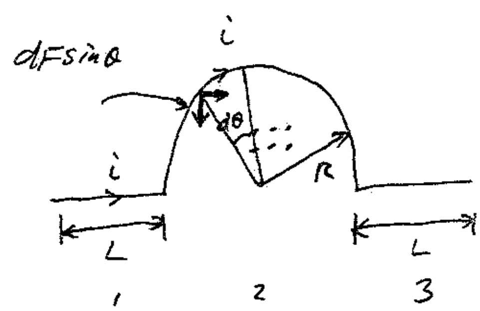


$$F_1 = F_3 = iLB$$

$$dF_2 = iB\,dL = iB\,R\,d\theta$$

$$F_2 = \int dF_2 \sin\theta = iBR \sin\theta\, d\theta$$

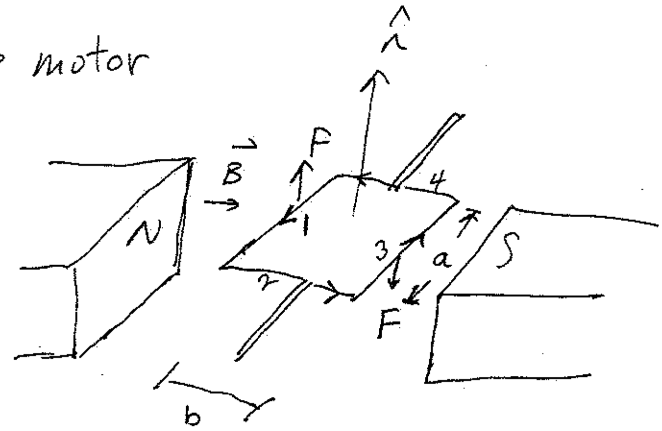
$$F_2 = \int_0^{\pi} dF \sin\theta = \int_0^{\pi} iBR \sin\theta\, d\theta = 2iBR$$

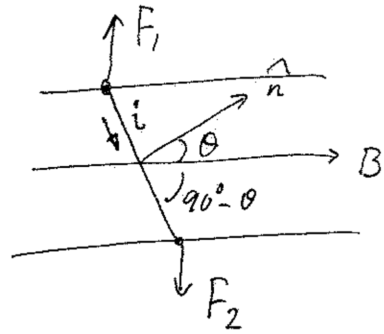
$$\therefore F_B = F_1 + F_2 + F_3 = 2iBR + 2LBi = 2iB(L+R)$$

10 Torque on a current loop.

① Magnetic force made up the working mechanism of electric motors.

② simple motor





$F_2 = ibB\sin(90° - \theta) = iBb\cos\theta$

$F_4 = -F_2$

$F_1 = iaB\sin 90° = iaB$

$F_3 = iaB$

$\tau' = (iaB\frac{b}{2}\sin\theta) + (iaB\frac{b}{2}\sin\theta)$

$= iabB\sin\theta$

for N turns $\rightarrow$ $\tau = N\tau' = NiabB\sin\theta$

$= (NiA)B\sin\theta$, $A = ab$ = area

11. Magnetic dipole.

— μ_B . for a current carrying coil.

→ electrons moving in th atoms,

$\mu \equiv NiA$.

∴ $\tau = \mu B \sin\theta$
$= \vec{\mu}_B \times \vec{B}$ $\Rightarrow U_B = -\vec{\mu}\cdot\vec{B}$.

Remember $\tau_E = \vec{\mu}_E \times \vec{E} \Rightarrow U_E = -\vec{P}_E \cdot \vec{B}$

① for a magnetic dipole to have lowest energy $\theta = 0$

$$U_B = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta = -\mu B.$$

With all th dipole line up with th magnetic field.

② for a magnetic dipole to have highest energy ; $\theta = 180°$

$$U_B = -\vec{\mu}\cdot\vec{B} = -\mu B \cos 180° = \mu B$$

$\Delta U = (+\mu B) - (-\mu B) = 2\mu B$, — work require to flip th dipole.