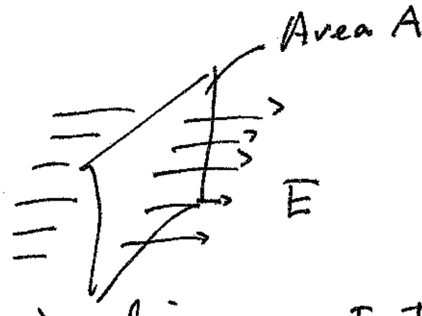
Chapter 24 Gauss's law

24.1 Electric flux

Electric field line : qualitatively

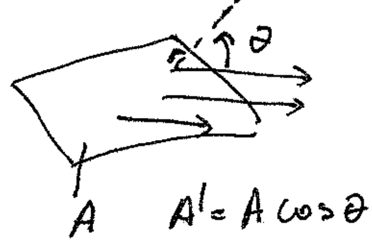
Electric flux : quantitatively



$\Phi_E \equiv EA$

= Electric flux

= proportional to the number of electric field lines penetrating some surface.



if the field lines do not perpendicular to the surface

$$\Phi_E = EA' = EA\cos\theta$$

$$= \vec{E}\cdot\vec{A} \qquad (\text{note: } \vec{A}\cdot\vec{B} = AB\cos\theta)$$

$$= E_i\,\Delta A_i \cos\theta_i = E_i \cdot \Delta A_i$$

in general

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i = \oint_{\text{Surface}} E \cdot dA = \oint E_n dA$$

$E_n \equiv$ Normal to the surface

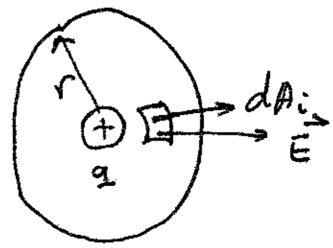
The sign of flux depends on the dot product of this integral.

24.2 Gauss's law –

– the relation between the net electric flux through a closed surface (called Gaussian Surface) and the charges enclosed by the surface.

$E \cdot \Delta A_i = E \Delta A_i$

$$\Phi_E = \oint E \cdot dA = \oint E\, dA = E \oint dA$$



But $E = k_e \frac{q}{r^2}$

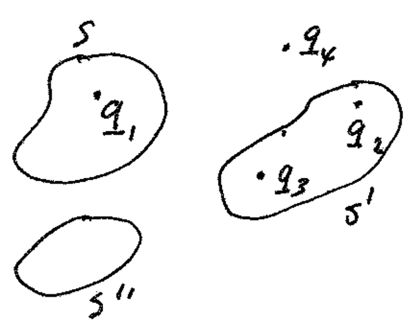
$$\Phi_E = k_e \frac{q}{r^2}(4\pi r^2) = 4\pi k_e q$$

But $k_e = \frac{1}{4\pi \epsilon_0}$

$$\therefore \boxed{\Phi_E = \frac{q}{\epsilon_0}}$$

We can choose a gaussian surface the cover the charge.
The net flux through any closed surface surrounding a point charge q is $\frac{q}{\epsilon_0}$ independent of the surface.

Many charges



$$\Phi_{total} = \oint E \cdot dA = \oint (E_1 + E_2 + \cdots) \cdot dA$$

Gauss's law $\quad \Phi_E = \oint \vec{E} \cdot \vec{A} = \frac{q_{in}}{\epsilon_0}$

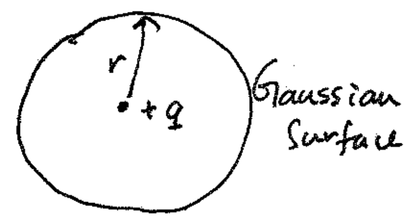
or. $\quad \epsilon_0 \Phi_E = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{in}$

Note: Zero flux is not zero field.

— Gauss's law states that the electric flux is proportional to the enclosed charge. Not the electric field.

24.3 Applications of Gauss's law

① Point charge



$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

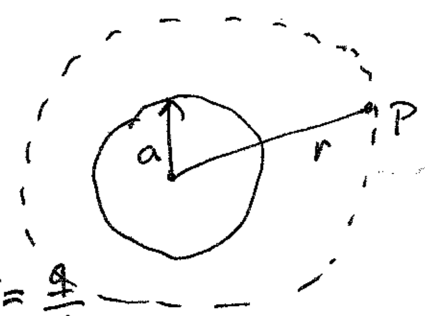
By symmetry E is constant on the surface

$$\therefore \Phi_E = E\oint dA = E\cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = k_e\frac{q}{r^2}$$

② A spherically symmetric charge distribution

②-1 : A point outside the sphere



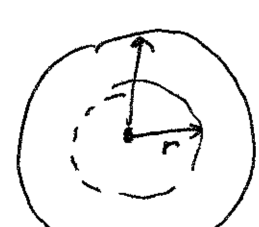
Pick a Gaussian surface bigger than the surface

$$\Phi_E = \oint E\cdot dA = E\oint dA = E\cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = k_e\frac{q}{r^2} \quad (r>a)$$

②-2. A point ~~and~~ inside the sphere.

Pick a Gaussian surface as shown



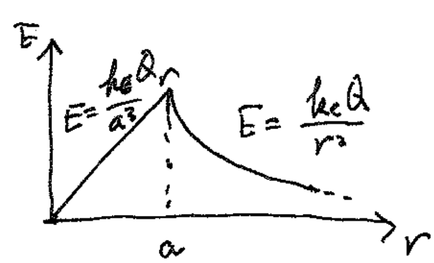
$$q_{in} = \rho V' = \rho\cdot\frac{4}{3}\pi r^3$$

$$\Phi_E = \oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q_{in}}{4\pi\epsilon_0 r^2} = \frac{\rho(\frac{4}{3}\pi r^3)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}r$$

$$\rho = Q/\tfrac{4}{3}\pi a^3$$

$$\therefore E = \frac{Qr}{4\pi\epsilon_0 a^3} = k_e\frac{Q}{a^3}r \quad (r<a)$$



③ Electric field due to a Thin Spherical ~~Sphere~~ Shell

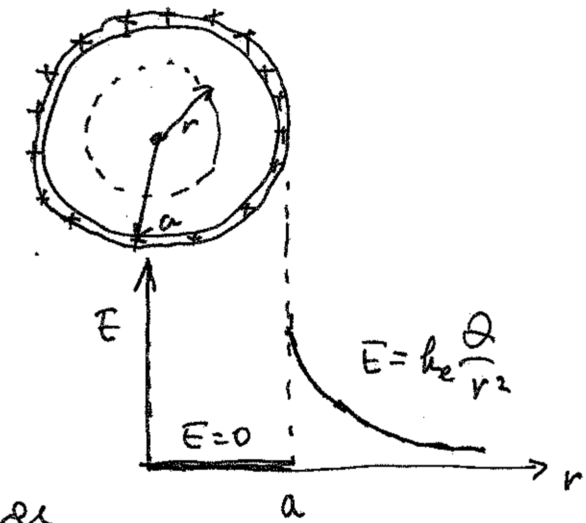
③-1 : Outside the Shell, $r > a$

$$E = k_e \frac{Q}{r^2}$$

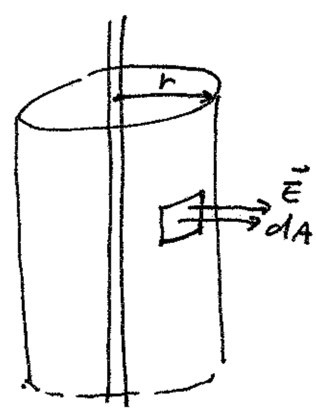
③-2 : Inside the Shell, $r < a$

The Gaussian Surface doesn't include any Charge inside it

$$\therefore \vec{E} = 0$$



④ A cylindrically symmetric Charge distribution.



$$\Phi_E = \oint E \cdot dA = E \oint dA$$

$$= EA = \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

$$A = 2\pi r l$$

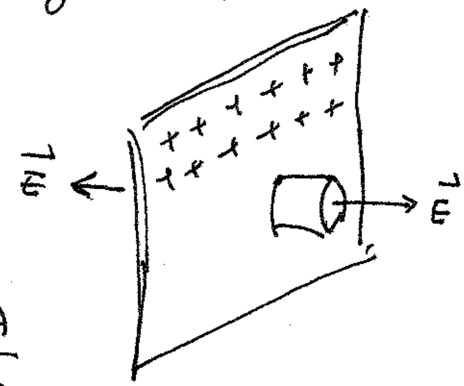
$$\therefore E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r} = 2k_e \frac{\lambda}{r}$$

Note: The electric field is symmetric But $E \sim \frac{1}{r}$

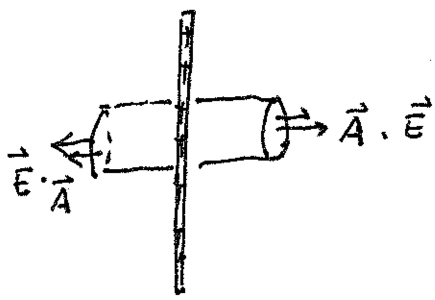
(5) A plane of Charge (thin)

Both side. The flux are positive



$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$



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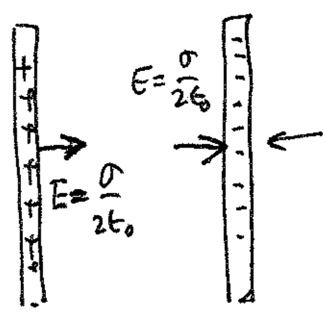


Therefore inside this two infinite planes the total $\vec{E}$ field

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

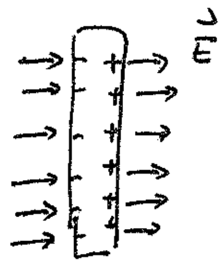
→ A practical way to obtain a Uniform field.

To Apply Gauss's law

1. The value can be argued by symmetry to be constant over the surface
2. The dot product $E \cdot dA$ can be expressed as simple algebraic product
3. The dot product $\Phi_E = \oint E \cdot dA = \frac{q_{in}}{\epsilon_0}$ can be zero if $E \perp dA$
4. The field can be argued to be zero over the surface.

24.4 Conductor in Electro Static Equilibrium (read page 750 carefully)

1). The electric field is zero everywhere inside the conductor
2) If an isolated conductor carries a charge, the charge resides on it's surface
3) The electric field just outside it surface $E = \frac{\sigma}{\epsilon_0}$
4) On an irregularly shaped conductor, the surface charge density is greatest at location where the radius of curvature of the surface is smallest. $E \sim \sigma \sim \frac{q}{r^2}$

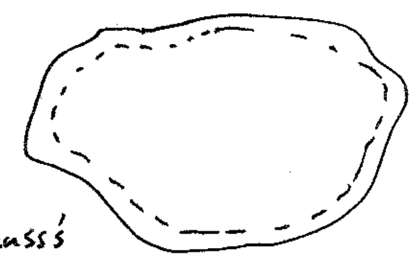


$E \equiv$ external

1) place in an external field as in fig 24.16 (page 750) for electro static equilibrium.
If not, free electrons in the conductor will experience an acceleration due to the external field.

2). The external field causes the separation of electrons and positive to create an electric field inside to oppose the external field ($\sim 10^{-16}$ second)

2)



due to 1) the field inside the conductor is zero. from Gauss's law, the charge inside the dashed Gaussian surface is zero
→ All the net charge must be on the surface.

3) If the E field is not perpendicular to surface

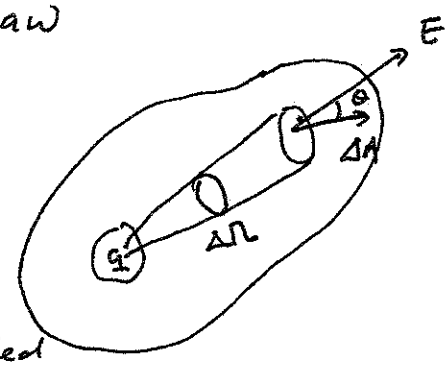
A $E_{\parallel}$ component will drive the charge. Then it will not be in electro static equilibrium.

$$\Phi_E = \oint E\,dA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$ on the surface

Check Fig 24.19 page 751

Do Example 24.10 page 752 (A sphere inside a spherical shell)

24.5 Derivation of Gauss's law



Solid angle $\Delta\Omega \equiv \frac{\Delta A}{r^2}$

[Steradian]
dimensionless

The total Solid angle subtended by a sphere $= \Omega = \frac{4\pi r^2}{r^2} = 4\pi$ Steradians.

for a Charge particle q

$$\Delta\Phi_E = E \cdot \Delta A = E\cos\theta\,\Delta A = k_e \frac{q}{r^2}\,\Delta A\cos\theta$$

$$\Phi_E = \oint k_e \frac{q}{r^2}\,\Delta A\cos\theta = k_e q \oint \frac{dA\cos\theta}{r^2} = k_e q \oint d\Omega = 4\pi k_e q$$

$$= \frac{q}{\epsilon_0}$$