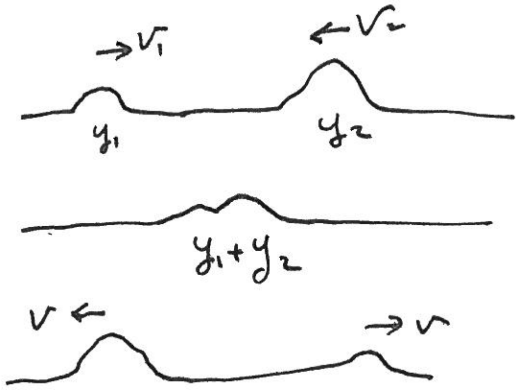
# Chap 18 Superposition and Standing waves



## 18.1 Superposition and interference

Superposition principle : If two waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave function

→ two traveling waves can pass each other without being destroyed or even altered.




$y_1 = A\sin(kx-\omega t)$ . $y_2 = A\sin(kx-\omega t+\phi)$

$y = y_1 + y_2 = A[\sin(kx-\omega t) + \sin(kx-\omega t+\phi)]$

$= 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)$

$k = \frac{2\pi}{\lambda}$ , $\omega = 2\pi f$

$\phi$ = phase angle

$\phi = 0$ $y = 2A\sin(kx-\omega t)$

$\phi = \pi$ $y = 2A\sin(kx-\omega t)$

$\phi = \frac{\pi}{2}$ $y = 2A\cos\left(\frac{\pi}{4}\right)\sin\left(kx-\omega t+\frac{\pi}{4}\right)$

$\sin A + \sin B$

$= 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$

Check Figure 18.4

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Therefore it is useful to express path difference $r$ in terms of phase angle.

$$\Delta r = \frac{\phi}{2\pi}\lambda = (2n)\frac{\lambda}{2} \quad \text{for constructive interference}$$

$$\Delta r = (2n+1)\frac{\lambda}{2} \quad \text{for destructive interference}$$

## 18.2 Standing wave


$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$= 2A\sin(kx)\cos(\omega t)$$

$\sin(a \pm b) = \sin A\cos B \pm \cos A\sin B$

- Standing wave
  - with a stationary outline
  - without $(kx - \omega t)$ terms, Not a traveling wave.
  - A special kind of simple harmonic oscillation. all the elements in a standing wave is doing SHM.

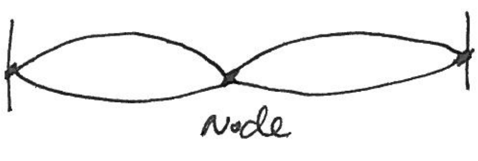
$$y = \underbrace{[2A\sin(kx)]}_{\text{Amplitude}}\ \underbrace{\cos\omega t}_{\text{Oscillation}}$$

$kx = n\pi$, $2A\sin kx = 0$ minimum Amplitude

$\frac{2\pi}{\lambda}x = n\pi$,

$x = \frac{n}{2}\lambda$, amplitude $= 0$

Called Node



Antinode – Maximum displacement occurs

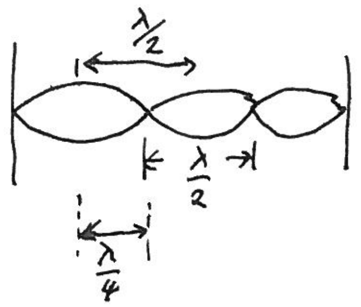
$$y = 2A \sin(kx) \cos\omega t$$

$$\sin kx = \pm 1$$

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

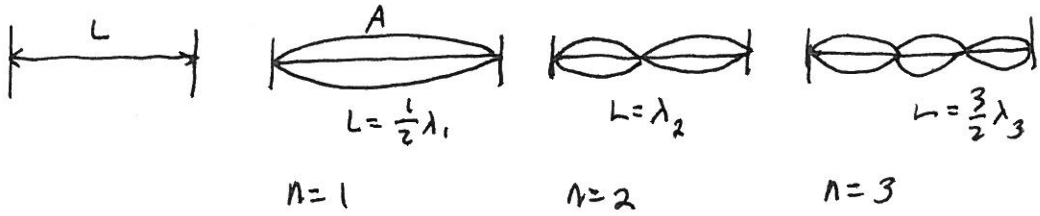
$$\frac{2\pi}{\lambda} x = \frac{\pi}{2}$$

$$\rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} = \frac{n\lambda}{4}, \; n = 1, 3, 5 \cdots$$




## 18.3 Standing waves in a String Fixed @ Both Ends

Due to the ends are fixed, the ends should have zero displacement and are nodes by definition.
→ The Boundary condition results in the string having a number of natural frequencies → Normal Modes




$$\lambda_n = \frac{2L}{n}, \; n = 1, 2, 3.$$

$$f_n = \frac{v}{\lambda} \quad \therefore f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}, \; n = 1, 2, 3 \cdots, \; v = \sqrt{\frac{T}{\mu}}$$

$$= \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$ – fundamental

The others are called "harmonic series"

Also, since $y(x,t) = 2A\sin(kx)\cos(\omega t)$

Boundary Conditions. $y(0,t) = 0$ — ①
$y(L,t) = 0$ — ②

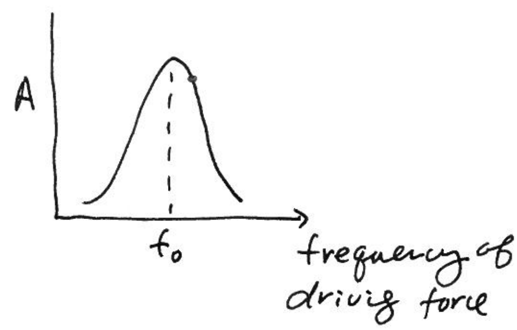
① $y(0,t) = \sin kx = 0$

② $y(L,t) = 0 \rightarrow \sin(kL) = 0$

$k_n L = n\pi$

$\left(\frac{2\pi}{\lambda_n}\right) L = n\pi \rightarrow \lambda_n = \frac{2L}{n}$

## 18.4 Resonance




If a periodic force is applied to a string, the amplitude of the resulting motion is at it greatest when the frequency of the applied force is equal to one of the natural frequency

→ Resonance

## 18.5 Standing waves in Air Column

Same as that of a transverse waves

## 18.7 Beats: Interference in time

(temporal Interference)

→ happens in two waves having similar frequencies.

$y_1 = A\cos\omega_1 t = A\cos(2\pi f_1 t)$

$y_2 = A\cos\omega_2 t = A\cos(2\pi f_2 t)$

$y = y_1 + y_2 = A\left(\cos 2\pi f_1 t + \cos 2\pi f_2 t\right)$

Note: $\cos A + \cos B = 2\cos\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)$

$$\therefore\ y = \underbrace{\left[2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t\right]}_{\text{Effective Amplitude}} \cos 2\pi\left(\frac{f_1+f_2}{2}\right)t$$

Check fig 18.22, page 565

Effective frequency $= \dfrac{f_1+f_2}{2}$

$$A_{resultant} = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t$$

When $\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t = \pm 1$ $\rightarrow$ Maximum Amplitude

$f_{beat} = |f_1 - f_2|$ $\rightarrow$ Beat frequency

## 18.8 Nonsinusoidal Wave Patterns

Fourier Series

$$y(t) = \sum_n \left(A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t\right)$$

Check Fig 18.25, P568