



## Solution

*Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables are not allowed. Please hand in both solution and this problem sheet. ABSOLUTELY NO CHEATING!*

### Problems (5 Problems, total 110%)

Note: in all problems, if a parameter is not given, you can assume it with the symbol you like.

- Electric charge:** (a) If we place a total charge  $Q$  (made up of small charges  $q$ ) on a solid conducting sphere. What will the charge distribution? In (a), if we have total charge  $Q$  place on a sphere of radius  $R$ . What will be the charge density? (20%)

**ANS:** (a) The charge will be evenly distributed on the surface of the sphere, since the sphere is a solid conductor, charges will repel one another. As a result, the charges will spread evenly on the surface. (b) Follow (a) the charges will be on the surface, and we will have a surface charge density of  $\frac{Q}{4\pi R^2}$

- Electric Power:** In a circuit powered by a battery with a potential difference  $\Delta V$ , the total loading of the circuit can be represented by a resistor loading of  $R$ . What is the power consumption of the resistor?

**ANS:** When the circuit is connected (ON), the battery provides the energy by pushing charge  $Q$  through the circuit passing the resistor. Let  $U$  be the electrical potential energy, the energy consumption rate is

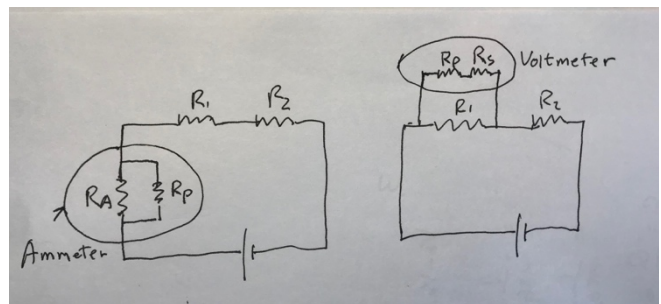
$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \Delta V \left(\frac{dQ}{dt}\right) = I\Delta V$$

Power is the energy consumption rate, and  $\Delta V = IR$ ; or  $\frac{\Delta V}{R} = I$ . Therefore,

$$P \equiv \frac{dU}{dt} = I\Delta V = I IR = I^2 R = \frac{(\Delta V)^2}{R}$$

- Measurements:** (20%) (a) As shown in the figure, an Ammeter is usually used to measure the current in the circuit and connected in series with the circuit.

The ammeter itself usually possess some internal resistance represented by the resistor  $R_A$ , typically  $60 \Omega$ . In addition, inside, the ammeter usually parallelly connect a small resistor of  $R_p$  with its resistor much less than  $R_p$ ; (b) however to measure the voltage, the voltmeter should be connected in parallel with the measured component. Inside the voltage, there is usually a large resistor connected to internal resistor in series. In both cases, explain why?

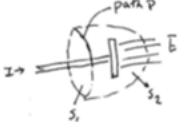


**ANS:** (a) The connection of a smaller resistor inside the ammeter is to ensure most of the measured

current will go through the smaller resistor (less power dissipation), so the measurement does not affect the total current in the circuit. (b) The connection of a larger resistor inside the ammeter is to ensure most of the measured current will NOT go through the larger resistor (less power dissipation), so the measurement does not affect the total current in the circuit. In both cases, the effects were tried NOT to affect the circuit due to measurement.

4. **Current in the capacitor:** (30%). The following figure schematically represents the current in a circuit involving capacitor. Between the two plates that are charged, there is no real current go through it; this make the discontinuity of the current. This problem is solved by Maxwell by inserting a term in the equation of Ampere's law. (a) What is Ampere's law for magnetism? (b) to make the current continuous, not discontinued, what is Maxwell's postulation? (c) Prove this term is indeed equivalent to the current  $I$ .

ANS:



1) Path P as boundary of  $S_1$   
Real current passes it

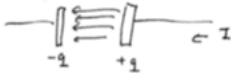
2) Path P as boundary of  $S_2$ .  
No conduction current passes  
→ Contradictory situation, due to the discontinuity.

Maxwell postulated an additional term in the Ampere's law called displacement current

$$I_d \equiv \text{displacement current} \equiv \epsilon_0 \frac{d\Phi_E}{dt} \quad \Phi_E = \int E \cdot dA$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

in Fig. 30.24



$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} = I \quad \text{Same as the conduction current } I.$$