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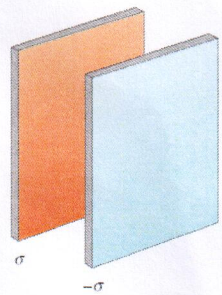
General Physics II, Midterm 2
PHYS1000AA, AB, AC, Class year 111-2
03-16-2023

SN:____________________, Name:____________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables ***are*** *allowed. Please hand in both solution and this problem sheet.*
ABSOLUTELY NO CHEATING!

Problems (Total 5 problems, 100%)

1. **Gauss law:** (20%) (a) What is Gauss Law? (b) Whatever your answer is in (a), prove it. Please assume all parameters you need.
2. **Dipole:** (total 35%) (a, 15%) Derive the electric field set up by a pair of positive and negative charges of charge $\boldsymbol{q}$. The charges are vertically standing separated by a distance $\boldsymbol{d}$ (in the z- axis direction). If we take the center of mass of the charges systems to be the origin of our coordinate system, and the vertical axis is the z-axis. What is the electric field intensity at a z distance from the origin that can be observed? (b, 10%) Now, if the dipole is making an angle f with respected to the z-axis, and an uniform electric field is applied perpendicular to the z-axis (from left to right) making an angle q with the dipole, What is the torque that can be generated by the electric field? (c, 10%) If this electric field is to flip the dipole from an initial angle q =90° to an angle q, how much work is needed?
3. /**Electric field of a solid conducting sphere:** (10%) A spherical solid conducting sphere has radius $\boldsymbol{a}$, and a total charge +Q. What is the electric field at distance 2a from the center of the sphere?
4. **Infinite planes:** (15%) Two infinite, non-conducting sheets of charge are parallel to each other, as shown on the right. The sheet on the left has a uniform surface charge density $\boldsymbol{\sigma}$, and the one on the right has a uniform charge density $-\boldsymbol{\sigma}$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.



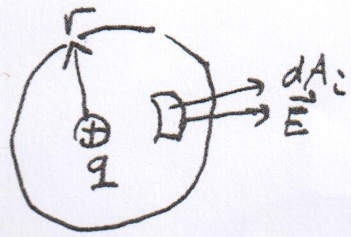
5. **Bohr model:** (20%) Bohr model of hydrogen atom assumes the electron of hydrogen atom of charge $\boldsymbol{q}$ is moving around the nucleus in a circular motion (classical picture). If the radius of hydrogen is $\boldsymbol{r} = 5.29\times10^{-11}$ m, (a) what is the strength of the force on the electron? (b) what is its centrifugal acceleration? (c) what is it orbital speed? Given $1/4\pi\epsilon_0 = 8.99\times10^9$ N•m^2/C^2. Electron has charge 1.6×10^{-19} C, and mass 9.11×10^{-31}kg.

1.

(a) Gauss law — the relation between the net electric flux through a closed surface and the charge enclosed by the surface

i.e. $\Phi_E = \frac{q}{\epsilon_0}$. $\Phi_E \equiv$ the electric flux

(b) Take a point charge for example



$\vec{E} \cdot dA_i = E \Delta A_i$

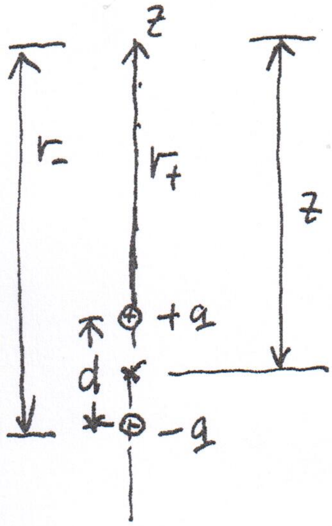
$\Phi_E = \oint E \cdot dA = \oint E dA = E \oint dA$

$\vec{E} = k_e \frac{q}{r^2}$

$\therefore \Phi_E = k_e \frac{q}{r^2} \cdot 4\pi r^2 = 4\pi \frac{1}{4\pi\epsilon_0} q$ $\therefore \Phi_E = \frac{q}{\epsilon_0}$

2. (a) This is the same problem as in the P23-6 of the lecture notes.

Refer to the figure to the right. The electric field is the vector sum of the two charges



$$\therefore \vec{E} = \vec{E}_{(+)} + \vec{E}_{(-)}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r_+^2} + \frac{-q}{r_-^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-\frac{1}{2}d)^2} + \frac{-1}{(z+\frac{1}{2}d)^2}\right]$$, Since z is a constant

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$ But $z \gg d$, $\frac{d}{2z} \ll 1$

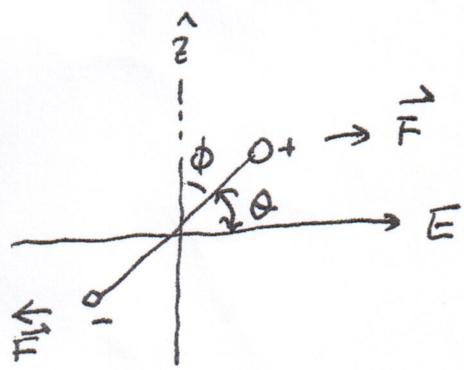
$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{2d}{2z}+\cdots\right) - \left(1-\frac{2d}{2z}+\cdots\right)\right]$$ So we can expand the polynominal

$$= \frac{q}{4\pi\epsilon_0 z^3}\left[1+\frac{2d}{2z}+\frac{2d}{2z}-1+\cdots\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{d}{z^3}$$ if we define $\vec{P} = qd$

Then $E = \frac{1}{2\pi\epsilon_0}\frac{\vec{P}}{z^3}$

(b) the forces are $+\vec{F}$ and $-\vec{F}$. But the both generate a clockwise torque



$$\tau = 2\cdot F\cdot\frac{d}{2}\sin\theta$$

$$= 2qE\frac{d}{2}\sin\theta = PE\sin\theta$$

2(c) The work needed is $U_f - U_i$

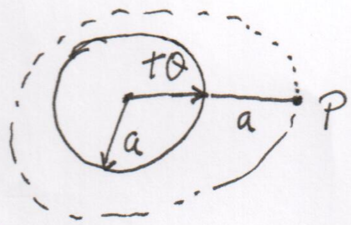
$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} pE \sin\theta \, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$= pE(\cos\theta_i - \cos\theta_f) \quad , \quad \theta_i = 90^\circ$$

$$\therefore U_f - U_i = \Delta U = -pE\cos\theta_f = -\vec{p}\cdot\vec{E}$$

this much energy is needed

3, We can construct a virtual sphere of radius 2a ~~t~~hat encloses the



Sphere, shown as the dotted circle in the figure above

Since the field is radial outward ~~and~~ and symmetric

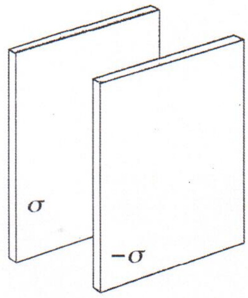
We can use Gauss law to find the field at point P.

$$\oint \vec{E}_p \cdot d\vec{A} = \frac{+Q}{\epsilon_0} \quad , \quad \vec{E}_p \cdot \vec{A} = \frac{+Q}{\epsilon_0} \quad E_p 4\pi r^2 = \frac{+Q}{\epsilon_0}$$

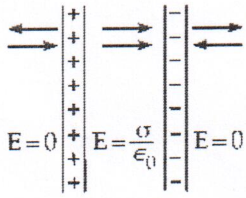
$$\therefore E_p = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad , \quad r^2 = (2a)^2 = 4a^2$$

$$\therefore E_p = \frac{1}{4\pi\epsilon_0}\frac{+Q}{4a^2}$$

4 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 24.8:



$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$.(See Textbook)



(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\mathbf{E} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\mathbf{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}.$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\mathbf{E} = \boxed{0}$

5. The electric force of the electron in the hydrogen atom is subject to the proton

(a) $F = \frac{1}{4\pi\epsilon_0} \frac{|q||q|}{r^2} = 8.99\times10^{9} N\cdot m^2/C^2 \cdot \frac{(1.6\times10^{-19})^2}{(5.29\times10^{-11} m)^2}$

$= 82.3\times10^{-9} N$

(b) $a = \frac{F}{m_e} = \frac{82.3\times10^{-9} N}{9.11\times10^{-31} kg} = 9.03\times10^{22} m/s^2$

(c) $a = \frac{v^2}{r}$; $v = (ar)^{\frac{1}{2}}$

$= (9.03\times10^{22} m/s^2 \times 5.29\times10^{-11} m)^{\frac{1}{2}}$

$= 2.19\times10^{6} m/s$

This is a classical picture of hydrogen atom. That agree with the Newton's law. this speed is about 1% of the speed of light, therefore Using Newton's 2nd law is appropriate.