國立東華大學
Department of Physics
National Dong Hwa University, 1, Sec. 2,
Da Hsueh Rd., Shoufeng, Hualien, 974, Taiwan

General Physics II, Final 2
PHYS1000AA, AB, AC, Class year 111-2
06-08-2023

SN:____________________, Name:____________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are NOT allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - .....$ Time average $\overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T}\int_0^T x(t)^n dt$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$ the general solution of this equation is $x(t) = x_0 \cos(at + \phi)$, where x_0 is the maximum, and ϕ is the phase angle.

$N_A = 6\times10^{23}$, R = Gas Constant= 8.31 J/mole K, room temperature =300K, 1atm=1.01×105 Pa.

$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}}$ for ideal gas. $\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$

Problems (5 Problems, total 100%)

1. **Single slit diffraction:** (20%) Light of wavelength 540 nm passes through a slit of width 0.200 mm. (a) The width of the central maximum on a screen is 8.10 mm. How far is the screen from the slit? (b) Determine the width of the first bright fringe to the side of the central maximum.

Solution: A single slit diffraction pattern, with the slit having width a, the dark fringe of order m occurs at angle θ_m, where $\sin\theta_m = m(\lambda/a)$ and $m = \pm1, \pm2, \pm3, \ldots$. The location, on a screen located distance L from the slit, of the dark fringe of order m (measured from $y = 0$ at the center of the central maximum) is

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda\left(\frac{L}{a}\right)$$

(a) The central maximum extends from the $m = +1$ dark fringe on one side to the $m = -1$ dark fringe on the other side, so the width of this central maximum is

$$\text{Central max. width} = (y_{\text{dark}})_{m=1} - (y_{\text{dark}})_{m=-1}$$
$$= (1)\left(\frac{\lambda L}{a}\right) - (-1)\left(\frac{\lambda L}{a}\right) = \frac{2\lambda L}{a}$$

Therefore,

$$L=\frac{a(\text{Central max. width})}{2\lambda}$$

$$=\frac{(0.200\times10^{-3}\text{ m})(8.10\times10^{-3}\text{ m})}{2(5.40\times10^{-7}\text{ m})}=\boxed{1.50\text{ m}}$$

(b) The first order bright fringe extends from the $m = 1$ dark fringe to the $m = 2$ dark fringe, or

$$(\Delta y_{\text{bright}})_1=(y_{\text{dark}})_{m=2}-(y_{\text{dark}})_{m=1}=2\left(\frac{\lambda L}{a}\right)-1\left(\frac{\lambda L}{a}\right)=\frac{\lambda L}{a}$$

$$=\frac{(5.40\times10^{-7}\text{ m})(1.50\text{ m})}{0.200\times10^{-3}\text{ m}}$$

$$=4.05\times10^{-3}\text{ m}=\boxed{4.05\text{ mm}}$$

Note that the width of the first order bright fringe is exactly one half the width of the central maximum.

2. <u>Snell's Law:</u> (20%) A plane sound wave in air at 20°C, with wavelength 589 mm, is incident on a smooth surface of water at 25°C at an angle of incidence of 13.0°. Determine (a) the angle of refraction for the sound wave and (b) the wavelength of the sound in water. (In this problem, you should derive the answer, plug in the numbers, but you don't have to carry out the final numbers. The speed of sound in water at 25.0°C is 1493 m/s.)

<u>Solution</u>:

(a) The law of refraction $n_1\sin\theta_1=n_2\sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1=\frac{c}{v_2}\sin\theta_2$$

$$\frac{\sin\theta_1}{v_1}=\frac{\sin\theta_2}{v_2}$$

In air at 20°C, the speed of sound is 343 m/s. The speed of sound in water at 25.0°C is 1493 m/s. The angle of incidence is 13.0°:

$$\frac{\sin 13.0°}{343\text{ m/s}}=\frac{\sin\theta_2}{1\,493\text{ m/s}}$$

$$\theta_2=\boxed{78.3°}$$

(b) The wave keeps constant frequency in all media:

$$f=\frac{v_1}{\lambda_1}=\frac{v_2}{\lambda_2}$$

$$\lambda_2=\frac{v_2\lambda_1}{v_1}=\frac{1\,493\text{ m/s}(0.589\text{ m})}{343\text{ m/s}}=\boxed{2.56\text{ m}}$$

3. **Faraday's law:** (20%) A coil formed by wrapping 50 turns of wire in the shape of a square is positioned in a magnetic field so that the normal to the plane of the coil makes an angle of 30° with the direction of the field. When the magnetic field is increased uniformly from 200 μT to 600 μT in 0.400 s, an emf of magnitude 80.0 mV is induced in the coil. What is the total length of the wire in the coil?

Solution:

Faraday's law, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$, becomes here

$$\mathcal{E} = -N\frac{d}{dt}\left(BA\cos\theta\right) = -NA\cos\theta\frac{dB}{dt}$$

The magnitude of the emf is

$$|\mathcal{E}| = NA\cos\theta\left(\frac{\Delta B}{\Delta t}\right)$$

The area is

$$A = \frac{|\mathcal{E}|}{N\cos\theta\left(\dfrac{\Delta B}{\Delta t}\right)}$$

$$A = \frac{80.0\times10^{-3}\text{ V}}{50\left(\cos 30.0^\circ\right)\left(\dfrac{600\times10^{-6}\text{ T}-200\times10^{-6}\text{ T}}{0.400\text{ s}}\right)} = 1.85\text{ m}^2$$

Each side of the coil has length $d=\sqrt{A}$, so the total length of the wire is

$$L = N(4d) = 4N\sqrt{A} = (4)(50)\sqrt{1.85\text{ m}^2} = \boxed{272\text{ m}}$$

4. **Displacement current:** (20%) An ideal capacitor has two conducting plate that separated by a distance ***L***, where two plates can have opposite charges (+***q*** and -***q***), in between, it is vacuum or stuffed with some non-conducting material called dielectric. However, in a capacitor, current can flow through this device. Maxwell inserted a term $I_d = \epsilon_0\frac{d\phi_E}{dt}$ to solve the problem. Prove this term is indeed current (has the unit of Coulomb/sec).

Solution:

From Gauss law, The term, $\phi_E = EA = (\frac{q}{\epsilon_0})$.

E is the electric field inside the capacitor, and ***A*** is the area of the capacitor

$I_d = \epsilon_0\frac{d\phi_E}{dt} = \epsilon_0\frac{d}{dx}\left(\frac{q}{\epsilon_0}\right) = \frac{dq}{dt} \equiv I$. So this term has the unit of a current.

5. **<u>Maxwell's Equations:</u>** (20%) Write down the 4 Maxwell's equations and briefly explain its meaning.

<u>Solution:</u>

Maxwell's equations

1. $\oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$ — You can calculate the electric field by constructing a Gaussian and the enclosed charge determines the field. this imply the possibility to isolate a single charge (positive or negative)

Gauss law Electricity

2. $\oint B\cdot dA = 0$ — this implies it is not possible to isolate a single pole of the magnet.

Gauss law magnetism

3 $\oint E\cdot ds = -\frac{d\Phi_B}{dt}$ the possibility to create an E field by a changing magnetic flux. this connects Electricity and magnetism

Faraday's law

4 $\oint B\cdot ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Ampere – Maxwell law — this determines the source of magnetis can be moving charge, or changing Electric flux overtime. and changing electric flux is a sort of current called displacement current.

6.

7.