

1. (a) For the wave function traveling to the right

$$y_1(x,t) = A \sin(kx - \omega t) \text{ to } +\vec{x} \text{ direction}$$

$$y_2(x,t) = A \sin(kx + \omega t)$$

(b) If a standing wave is created, it is formed with  $y_1$  and  $y_2$ . Then the wavefunction will be

$$Y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

Using the formula given

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta)$$

$$\cdot \cos \frac{1}{2}(\alpha - \beta)$$

$$Y = 2A \sin kx \cdot \cos \omega t \quad \leftarrow \text{this is not a traveling wave any more.}$$

(c) determine the node positions. It is a standing wave.

$$\therefore Y = \underbrace{2A \sin kx}_{y_1 + y_2} \cdot \underbrace{\cos \omega t}_{\text{oscillation with time } t}$$

↑  
oscillation with positions  $x$ .

To have nodes,  $\sin kx = 0$

or  $kx = n\pi$ , but  $k = \frac{2\pi}{\lambda}$

$$\frac{2\pi}{\lambda} x = n\pi \rightarrow x = n \frac{\lambda}{2}, n = 1, 2, 3, \dots$$

→ We need to know which harmonic to determine the wave length  $\lambda$

2.

Initial kinetic energy

$$E_{ki} = \frac{1}{2} m v_i^2$$

(a)

Total energy =

$$\frac{1}{2} m v_i^2 - \frac{GMEm}{R_E} = - \frac{GMEm}{r_{max}}$$

$$\Rightarrow v_i^2 = 2GM_E \left( \frac{1}{R_E} - \frac{1}{r_{max}} \right)$$

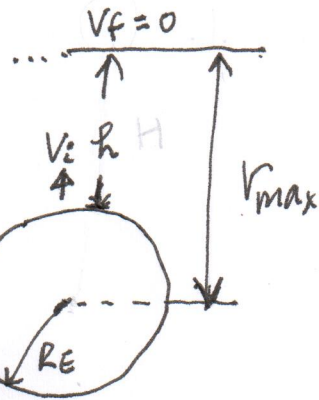
(b) if  $r_{max} = \infty \rightarrow \frac{1}{r_{max}} = 0$

then

$$v_i^2 = 2GM_E \left( \frac{1}{R_E} \right)$$

$$= v_{esc}^2$$

$$\therefore v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$



If we plug in numbers

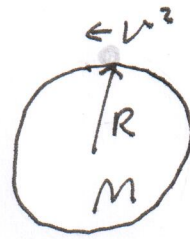
$$v_{es} \approx 10 \text{ km/sec}$$



3. The magnitude of the gravitational force exerted by the planet on an object

of mass  $m$  at the surface

is  $F = \frac{GMm}{R^2}$



let  $M$  = mass of the planet

$m$  = mass of the object

$= \frac{mv^2}{R}$  = Centrifugal force

$R$  = Earth radius

(a) If the gravitational force is less than the centrifugal force, then the object will fly off.

(b) But  $M = \frac{4}{3}\pi R^3 \rho$ ,  $\rho$  = the density

$v = \frac{2\pi R}{T}$ ,  $T$  = the period of the revolution

Plug in  $M$  and  $v$  into the equation

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{G \frac{4}{3}\pi R^3 \rho}{R^2} = \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R}$$

$$\Rightarrow T = \sqrt{\frac{3\pi}{G\rho}}$$

(c) Plug in numbers.  $G = 6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \cdot \text{kg}$

$\rho = 3 \times 10^3 \text{ kg/m}^3$

$T = \sqrt{\frac{3\pi}{G\rho}} \approx 6.86 \times 10^4 \text{ s} = 1.9 \text{ hour}$  No planet is spinning so fast

4.

The internal energy change

$$\Delta E_{\text{int}} = \Delta(n C_v T) = C_v \Delta(nT) \quad \text{--- (1)}$$

But  $PV = nRT$

↑ when  $n = \text{constant}$

$$\rightarrow nT = \frac{PV}{R} \quad \text{--- (2)}$$

Put (2) into (1)

$$\Delta E_{\text{int}} = C_v \Delta \left( \frac{PV}{R} \right)$$

But here  $P, V, R$  are all constant

$$\therefore \Delta E_{\text{int}} = 0$$

There is no change in the internal energy.

But in this cabin some air are leaving the cabin through openings.

The  $n$  did not stay constant

and the wood burning is lost through the air escaping the room



5 Consider an ideal gas goes  $V_i T_i \rightarrow V_f T_f$

From the thermodynamic (1<sup>st</sup>) law

$$dE_{\text{int}} = dQ + dW$$

$dW \equiv$  work done on the gas.

$$dQ = dE_{\text{int}} - dW$$

$$= -P dV$$

$$= dE_{\text{int}} + P dV$$

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$dQ = nC_v dT + \frac{nRT}{V} dV$$

$$\therefore \frac{dQ}{T} = nC_v \frac{dT}{T} + nR \frac{dV}{V}$$

$$\therefore \int_i^f \frac{dQ}{T} = \text{total entropy change}$$

$$= \int_{T_i}^{T_f} nC_v \frac{dT}{T} + \int_{V_i}^{V_f} \frac{nR \cdot dV}{V}$$

$$= nC_v \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$