



SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

The followings are some useful mathematics you may use without proof in answering your problems.

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots \cdot \text{Time average } \overline{x(t)^n} = \langle x(t)^n \rangle = \frac{1}{T} \int_0^T x(t)^n dt$$

For a second order differential equation, $\frac{d^2x}{dt^2} + ax = 0$ the general solution of this equation is

$$x(t) = x_0 \cos(at + \phi), \text{ where } x_0 \text{ is the maximum, and } \phi \text{ is the phase angle.}$$

$N_A = 6 \times 10^{23}$, $R = \text{Gas Constant} = 8.31 \text{ J/mole K}$, room temperature = 300K, 1atm = $1.01 \times 10^5 \text{ Pa}$.

$$\overline{v_x} = \frac{v_{rms}}{\sqrt{3}} \text{ for ideal gas. } \sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2}(\alpha \pm \beta) \cos \frac{1}{2}(\alpha \mp \beta)$$

Problems (5 Problems, total 120%)

- 1. Standing wave (20%):** To generate a standing wave between two vertical walls separate by a distance L , you set up a wave from the left wall traveling with a speed to the right, and from the right wall traveling with the same speed to the left. It is possible to generate a standing wave between the two walls. (a) Write down the wave functions for both the right-traveling and left-traveling waves. (b) Derive the wave function to describe the standing wave. (c) What are the conditions (or the x -position) to have a node?
- 2. Escape velocity: (20%)** If the radius of the earth is RE , the mass of our earth is ME . You through a stone of mass m with initial velocity Vi vertical up and reach a maximum height of H . (a) What is the initial velocity? (b) if you wish to through the stone vertically up so it will reach infinite, what is the initial velocity of the stone should be?
- 3. Gravitational Force: (30%)** Let the mass of a planet be M , radius R . The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. (a) Why (10%)? (b) Show that the corresponding shortest period of rotation is $T = \sqrt{\frac{3\pi}{G\rho}}$. Where ρ is the uniform density of the spherical planet? (10%) (c) Calculate the rotation period assuming a density of $3.0 \times 10^3 \text{ kg/m}^3$, typical of many planets, satellites, and asteroids. No astronomical object has ever been found to be spinning with a period shorter than that determined by this analysis. (10%). $G = 6.86 \times 10^{-11} \text{ m}^3/\text{s}^2\text{kg}$

4. **Thermodynamic 1st law: (30%)** A cabin of volume V is filled with air (which we consider to be an ideal gas) at an initial low temperature T_1 . After you light up a wood stove, the air temperature increases to T_2 . What is the resulting change in internal energy (E_{int}) of the air in the cabin? Hint: In this cabin, the inside should always maintain the same air pressure since the cabin is not air tight

5. **Entropy change: (20%)** What is the total entropy change for an ideal gas from initial volume and temperature V_i, T_i to final volume and temperature V_f, T_f ?