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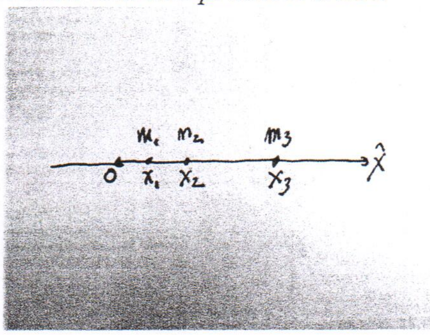
General Physical I, Midterm 1
PHYS1000AA, AB, AC, Class year 110-1
11-09-2021

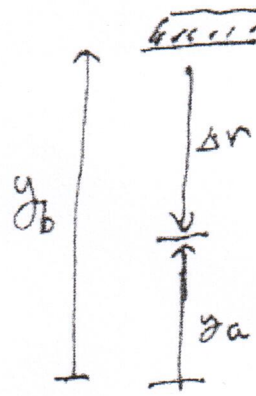
SN:____________________, Name:____________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables ***are*** *allowed. Please hand in both solution and this problem sheet.*
ABSOLUTELY NO CHEATING!

Problems (20% each, total 6 problems, 120%)

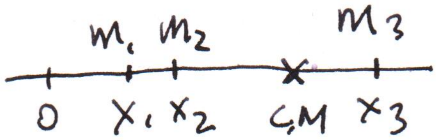


1. **Center of Mass:** (20%) As shown in the figure to the right, a system of three masses ($\mathbf{m_1}$, $\mathbf{m_2}$, and $\mathbf{m_3}$) located at the x-axis position ($\mathbf{x_1}$, $\mathbf{x_2}$, $\mathbf{x_3}$), respectively. If $\mathbf{m_3}$ is much larger than both $\mathbf{m_1}$, $\mathbf{m_2}$, where will be the center of mass in terms of parameters given?



2. **Conservation of energy: (20%)** An object of mass ***m*** is raised to a height, as shown in the figure to the right, and falls down to the floor. At the lowest position, what is its change in kinetic energy? (b) What is its change in potential energy? (c) What is its total energy change?
3. **Momentum conservation: (20%)** Linear momentum conservation is a consequence of Newton's 3rd law. Now we have two rigid particles collides head on. The two particles have masses $\mathbf{m_1}$ and $\mathbf{m_2}$, and after collision the velocities are $\mathbf{V_1}$ and $\mathbf{V_2}$, respectively. Show that the momentum after collision is a constant.
4. **Rotational moment of inertia: (20%)** Prove the rotational inertia of an annular cylinder is $\frac{1}{2}M(R_1^2 + R_2^2)$. The hollow cylinder has outer diameter $\mathbf{R_2}$ and inner diameter $\mathbf{R_1}$, length **L** and the total mass is **M**; assuming it has a constant density ρ.
5. **Mass-Spring system: (20%)** In a horizonal mass-spring system, a mass **m** is attached to a spring with force constant **k**. If you pull the mass with a force **F**, the spring has a displacement **x**, and release it. (a) Describe the motion of the mass, (b) Assume this is a perfect elastic spring, at what position the mass has its highest velocity?

1. Center of mass



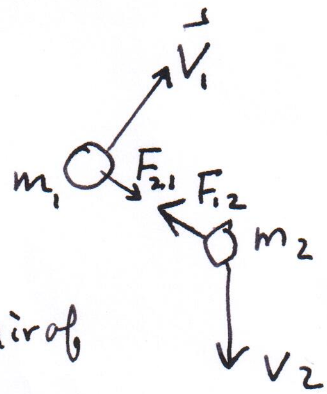
Since M3 is much larger than both m_1 and m_2,

the center of mass should be located more closer to m_3

Therefor $(m_1+m_2+m_3)X_{cm}=m_1X_1+m_2X_2+m_3X_3$

$$X_{CM}=\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}$$

3. Momentum Conservation



Shown in the figure. the two particles collided and move

During the collision, a pair of action-Reaction forces are the same but opposite in direction. i.e. $F_{12}=-F_{21}$

$\rightarrow F_{12}+F_{21}=0 \quad m_1a_1+m_2a_2=0$

$$\therefore m_1\frac{dV_1}{dt}+m_2\frac{dV_2}{dt}=0$$

$$\rightarrow \frac{d(m_1V_1)}{dt}+\frac{d(m_2V_2)}{dt}=0 \rightarrow \frac{d}{dt}(m_1V_1+m_2V_2)=0$$

Means. $\underbrace{m_1V_1}_{P_1}+\underbrace{m_2V_2}_{P_2}=\text{Constant}. \rightarrow$ Conservation of momentum

General Physics I, Midterm Exam 1 solution

2.

$$W_{\text{on book}} = (mg)\cdot\Delta r$$
$$= (-mg\hat{j})\cdot[(y_b - y_a)\hat{j}]$$
$$= mgy_b - mgy_a$$
$$= \Delta E_K$$
$$= \Delta K_{\text{book}}$$

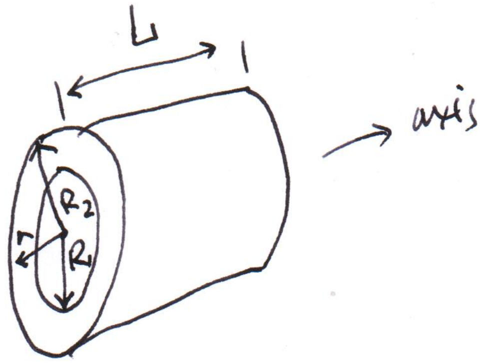
$$mgy_b - mgy_a = -(mgy_a - mgy_b)$$
$$= -(U_f - U_i)$$
$$= -\Delta U_g$$

$$\therefore\ \Delta K = -\Delta U_g$$

$$\Delta K + \Delta U_g = 0$$

4.

Annular Cylinder



$$I = \int dm\, r^2$$

$$dm = 2\pi r L \rho\, dr \qquad \rightarrow M = \pi R_2^2 L\rho - \pi R_1^2 L\rho$$

$$= \pi L \rho (R_2^2 - R_1^2)$$

$$\therefore I = \int 2\pi r L \rho\, r^2 dr$$

$$= 2\pi L\rho \int_{R_1}^{R_2} r^3 dr$$

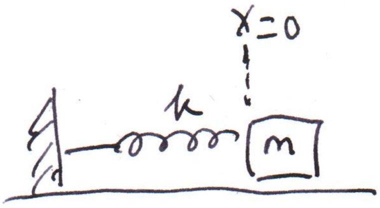
$$= 2\pi L \rho\, \frac{1}{4}(R_2^4 - R_1^4)$$

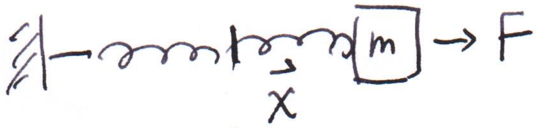
$$= 2\pi L\rho\, \frac{1}{4}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

Plug in $M = \pi L \rho (R_2^2 - R_1^2)$

$$I = \frac{1}{2} M (R_1^2 + R_2^2)$$

5.





(a) The mass will have an oscillation between $+x$ and $-x$
If no friction betwee the contact of mass and the floor.

(b) When $x=0$ the spring is at rest
When pull to $+x_{max}$ position. the spring has $\frac{1}{2}kx_{max}^2$ potential energy.
When the mas moves back to $x=0$ position,
All the energy $(\frac{1}{2}kx_{max}^2)$ Will transform to the kinetic energy of the mass

$$\therefore \quad \frac{1}{2}mV_{max}^2 = \frac{1}{2}kx_{max}^2$$