

Final-I Solution

1. Solution : (Similar to 15.4, text book 8th edition)

(a) For a spring, $F = -kx = ma$

$$a = \frac{d^2x}{dt^2} = \frac{-kx}{m}$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = \omega^2x, \text{ where } \omega = \sqrt{\frac{k}{m}}$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

(b) Here, $s = L\theta$

$$\text{So, } F = ma = m \frac{d^2s}{dt^2} = -mg \sin \theta$$

$$\frac{d^2s}{dt^2} = -g \sin \theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \sin \theta \quad \text{Since } s = L\theta$$

$$(c) \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta, \text{ if } \theta \text{ is very small, } \sin \theta = \theta$$

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \text{ where } \omega = \sqrt{\frac{g}{L}}$$

Which is a simple harmonic motion equation

2. Solution: (Similar to 18.2, text book 8th edition)

(a) The wave function of the travelling waves are

$$y_1 = A \sin(kx - \omega t) \quad (\text{for right})$$

$$y_2 = A \sin(kx + \omega t) \quad (\text{for left})$$

(b) Wave function for the standing wave will be

$$Y = y_1 + y_2$$

$$= A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$= A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$= 2A \sin kx \cos \omega t \quad \text{Since } \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{D-C}{2}$$

(c) The condition for the node can be found for $2A \sin kx = 0$

$$\text{or, } \sin kx = 0 \quad \text{or, } kx = n\pi$$

$$\frac{2\pi x}{\lambda} = n\pi, \quad \text{or, } x = \frac{n}{2} \lambda, \text{ where } n = 0, 1, 2, 3, \dots, N$$

$$\text{So, } x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots \text{ are the condition for NODE}$$

3. Solution: (Similar to page 472 , Halliday Book)

The internal energy change

$$\Delta E_{\text{int}} = \Delta(nC_v T) = C_v \Delta(nT) \dots \dots \dots (1)$$

But we know , $PV = nRT$ for a ideal gas

$$\text{So , } nT = \frac{PV}{R}$$

Put this value in equation (1) , we get

$$\Delta E_{\text{int}} = C_v \Delta\left(\frac{PV}{R}\right) , \text{ But here } P, V, R \text{ are all constant}$$

So there will no change of internal energy

$$\Delta E_{\text{int}} = 0$$

4. Solution: (Similar to 21.5 , text book 8th edition)

(a) C_p needs to consider extra work done by (or be done to) the gas ,
so extra energy is required.

(b) We know , $\Delta E_{\text{int}} = nC_v dT = Q + W = 0 - PdV$ (for adiabatic process)

$$nC_v dT = -PdV \text{ or, } ndT = -\frac{PdV}{C_v}$$

Now for a ideal gas, $PV = nRT$

Using differentiation we can write ,

$$PdV + VdP = nRdT$$

$$PdV + VdP = -\frac{PdV}{C_v} R = -\frac{(C_p - C_v)PdV}{C_v} , \text{ Since } R = C_p - C_v$$

$$PdV \left[1 + \frac{(C_p - C_v)}{C_v} \right] = -VdP$$

$$\frac{dV}{V} (1 + \gamma - 1) + \frac{dP}{P} = 0 , \text{ Since } \frac{C_p}{C_v} = \gamma$$

Using integration we get, $\ln P + \ln V^\gamma = \ln C$, Where C is constant

$$\therefore PV^\gamma = \text{Constant}$$

5. let two waves beig

$$y_1 = A \sin(kx - \omega t)$$

$$y_2 = A \sin(kx + \omega t)$$

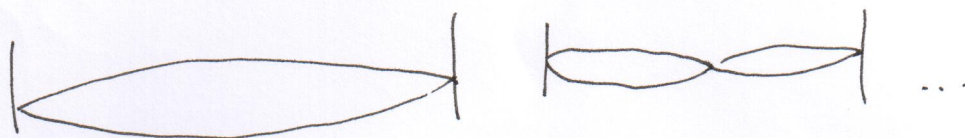
A is the amplitude

k = wave number

ω = angular frequency



⇓ After interaction



$$y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$= \underline{2A \sin(kx)} \cos(\omega t)$$

Use $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$

The new wave is no longer a traveling wave, it oscillates at ω frequency and the new amplitude is $2A \sin(kx)$, i.e. a function of x

When $kx = n\pi$, $2A \sin(kx) = 0$ it has minimum amplitude

When $\frac{2\pi}{\lambda} x = n\pi$

$x = \frac{n}{2} \lambda$, amplitude is zero

Called node.

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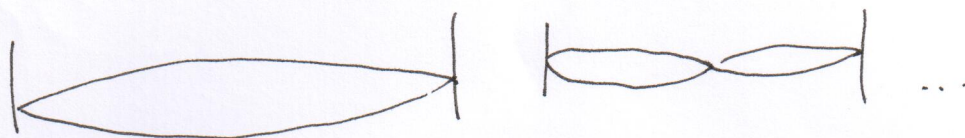
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