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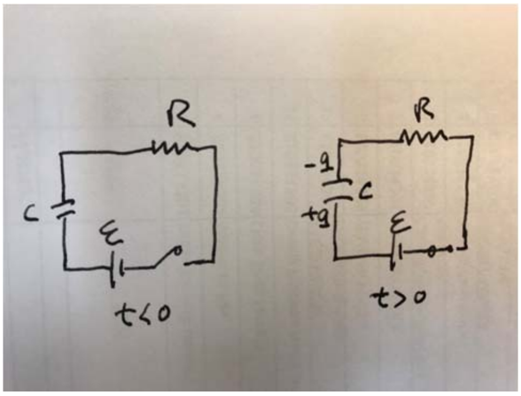
General Physics II, Midterm 3
PHYS10000AA, AB, AC, Class year 108
05-28-2020

SN:__________________, Name:_______________

ABSOLUTELY NO CHEATING!

Problems (4 Problems, total 100%)

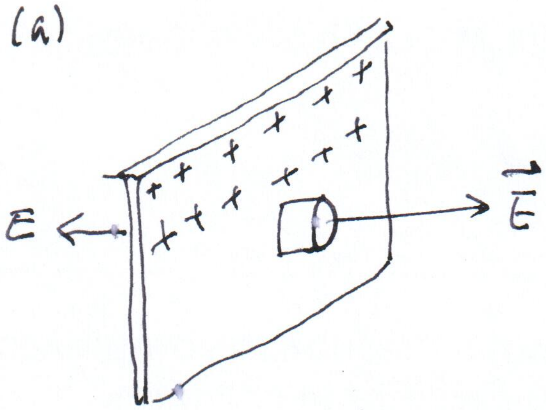
1. **A thin plane of charges:** (40%) (1) Suppose we have a large thin plane (with area ***A***) with uniform positive charges distributed on the surface its charge density is σ, Use Gauss law to determine the electric field on the surface both the magnitude and the direction. (2) If we bring another large plane with negative charge but the same charge density close to each other separated by a distance d, what is the final electric field inside the two infinite planes? (3) In (2), we form a parallel plate capacitor. What is the (electric) potential difference between these two plates? What is its capacitor? (4) IF now we insert a dielectric material, with dielectric constant κ, what is the new electric field inside the plates? What is the induced charge density near the negative plate?
2. **Electric field of a charged sphere:** (20%) What is the electric field of a spherically symmetric charged sphere of total charge Q? (1) At a distance **r** outside the sphere from the center, (2) at a distance r from the center inside the sphere. (3) Plot the electric field as a function of distance from the center of the sphere, assuming the radius of the sphere is R.
3. **RC circuit:** (20%) In the figure to the right, we have a RC circuit. If at ***t***=0 we turn the circuit, the capacitor start to charge. (1) What is the instantaneous current what the circuit is turned on at ***t***=0? (2) When the charging is complete, what is the total charge we can have at the capacitor? (3) Drive the differential equation that describe the charge as a function of time, ***q(t)***.



4. **Ampere's law:** (20%) (1) Write down what is ampere's law? (2) In a long current- carrying wire, if the current is ***I***, the diameter of the wire is ***R***. What is the magnetic field outside the wire, ***r***>***R***? (3) What is the magnetic field inside the wire when ***r***<***R***? (4) Plot the magnetic field as a function of ***r***.

1.

(a)



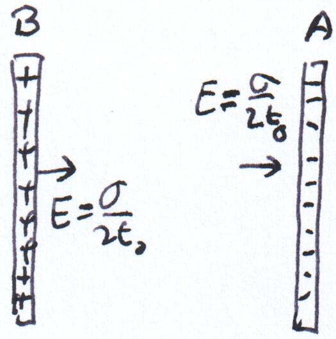
Let's make a gauss surface that is parallel to the plane surface

Gauss says

$$\Phi_E = 2\vec{E}\cdot\vec{A} = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$\therefore \vec{E} = \frac{\sigma}{2\epsilon_0}$, Normal to the surface of the plane

(b) Now we bring two of these planes, one positive and one negative plane together, separated by a distance



From the figure to the left. we see the total electric field is the sum of the two field by positive and negative plane

Therefore $E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

direction: to the right from the positive plane

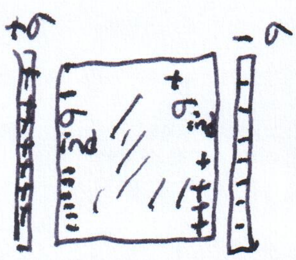
(c) Use the above figure, in general.

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s} = \Delta V = -\int_A^B E\,ds = -Ed$$

For this capacitor $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A} \;;\; C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}} = \frac{\epsilon_0 A}{d}$$

(d)



After inserting the dielectric material

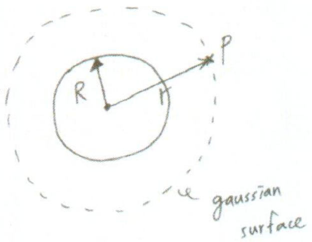
$$E = E_0 - E_{ind} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{ind}}{\epsilon_0} = \frac{\sigma}{K\epsilon_0}$$

K is the dielectic constant

$$\sigma_{ind} = \left(\frac{K-1}{K}\right)\sigma$$

2.

(1) A point outside the sphere : Pick a gaussian surface bigger than the surface of the sphere



$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}}$$

$$\Rightarrow E\oint dA = E\cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\cdot\frac{Q}{r^2}$$

$$= k_e\frac{Q}{r^2}\ (r>R) \qquad \propto \frac{1}{r^2}$$

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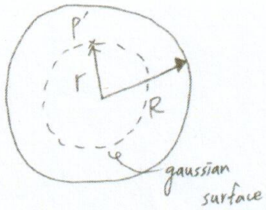
(2) A point inside the sphere : Pick a gaussian surface as shown



$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}}$$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0}, \quad q_{in} = \rho\cdot V$$

$$= \frac{Q}{\frac{4}{3}\pi R^3}\cdot\frac{4}{3}\pi r^3$$

$$= Q\,\frac{r^3}{R^3}$$

$$\therefore E = \frac{1}{4\pi\epsilon_0}\,\frac{Q}{R^3}\,r$$

$$= k_e\frac{Q}{R^3}\,r\ (r<R) \qquad \propto r$$

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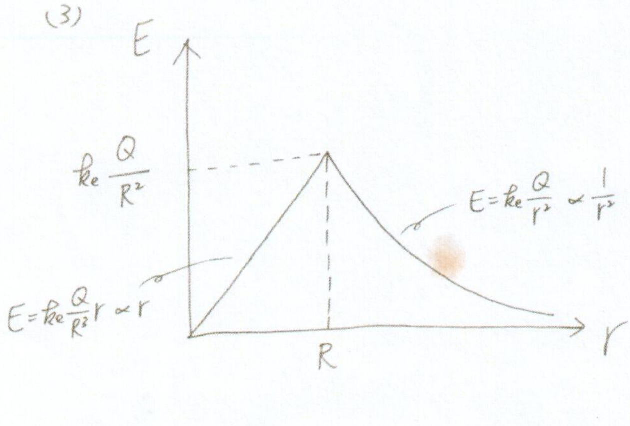
(3)



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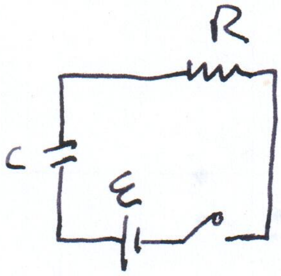
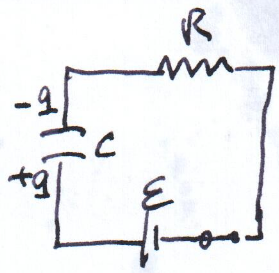


$t<0$



$t>0$

(1) at $t=0$. the Switch is turned on, the Capacitor is not Charge. $I_0 = \frac{\varepsilon}{R}$

(2) when the changing is Complete, there will be no current, therefore the $Q = C\varepsilon$

(3) $I = \frac{dq(t)}{dt}$ by definition

Use kirchhoff's law. we have

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$I = \frac{\varepsilon}{R} - \frac{q(t)}{RC}$$

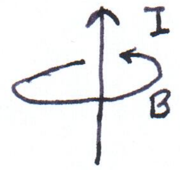
$$\frac{dq(t)}{dt} = -\frac{1}{RC} q(t) + \frac{\varepsilon}{R}$$

Or $$\underline{\frac{dq(t)}{dt} + \frac{1}{RC} q(t) - \frac{\varepsilon}{R} = 0}$$

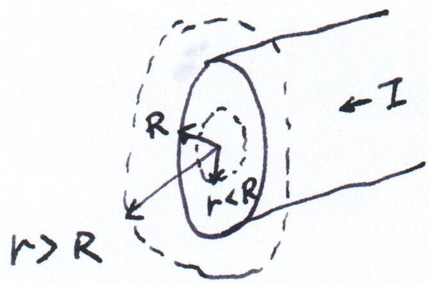
Solve this differencial equatin we can get the Charge as a function of tim, $q(t)$

4. Ampere's law

(1) $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$, I is the current enclosed by the Ampere loop chosen, such as



(2)



$r > R$

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad , \quad B \propto \frac{1}{r}$$

(3) when $r < R$

$$\oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi r = \mu_0 I' \quad , \quad I' = \left(\frac{\pi r^2}{\pi R^2}\right) I$$

$$\therefore B = \frac{1}{2\pi r} \cdot \mu_0 \cdot \frac{\pi r^2}{\pi R^2} I$$

$$= \mu_0 \frac{r}{R^2} I$$

$$= \frac{\mu_0 I}{R^2} r \qquad B \propto r \text{, when } r < R$$

(4)