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General Physics I, Final 2
PHYS10000AA, AB, AC, Class year 108
06-18-2020

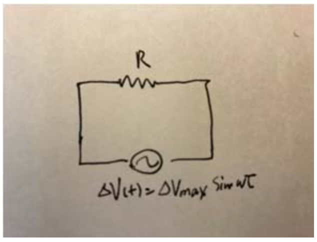
SN:____________________, Name:________________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

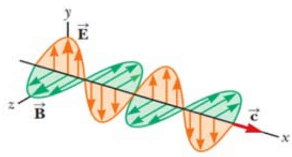
Problems (5 Problems, total 120%)

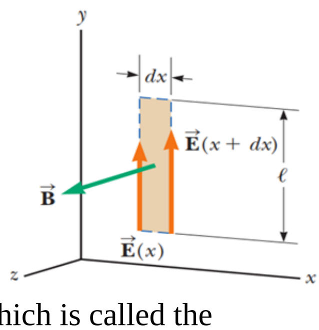
1. **AC source:** (30%) Refer to the figure to the right. Consider a very simple AC circuit, the AC source of voltage can be expressed as $\Delta V(t) = \Delta V_{max} \mathrm{Sin}\omega t$, and only a resistor with resistance ***R*** is connected in the circuit. (a)What is the maximum current that can be passed through the resistor? (b) Is there any phase change between the voltage source and the current? (c) What is the power generated by the resistor as a function of time? (d) What is the average (root mean square) power generated by the resistor?



2. **Maxwell equations:** (20%) Write down the four Maxwell's equations, and briefly explain what each equation means.

3. **Maxwell equations:** (20%) Consider an EM wave traveling in the x-axis direction as shown in the figure to the right. This describes the orthogonal relation of the oscillating Electricity (E) and magnetic (B) fields when the EM wave is traveling. Based on the figure, derive the follow relation between the ***E***(x, t) and ***B***(x, t) fields is: $\frac{\partial E(x,t)}{\partial x} = -\frac{\partial B(x,t)}{\partial t}$



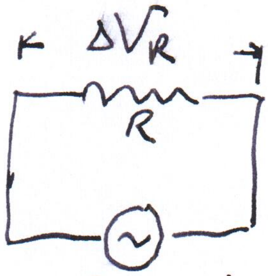


4. **EM wave energy:** (30%) The rate of flow of energy in an EM wave is defined as $\boldsymbol{S} \equiv \frac{1}{\mu_0} \boldsymbol{E} \times \boldsymbol{B}$, where ***S***, ***E***, ***B*** are all vectors, and ***S*** is named Poynting vector, it has unit W/m^2, is the rate of flow of energy through a surface area. (a) What is the instantaneous rate of energy in a plane wave that energy is passing through a unit area? (b) What is the time average of ***S***, which is called the intensity of the EM wave ***I***?

5. **Extra credit: Single slit diffraction:** (20%) A parallel beam of blue light (wavelength 420 nm) is incident on a small aperture. After passing through the aperture, the beam is no longer parallel but diverges at 1° to the incident direction. What is the diameter of the aperture? Note: for small angle θ, $\sin\theta \cong \theta$. (15%).

1.

(a) $\Delta V(t) = \Delta V_{max} \sin \omega t$



$\Delta V(t) = \Delta V_{max} \sin \omega t$

Refer to the figure to the right, the Voltage across the resistor is the same as that of the Source,

$$\therefore \Delta V = \Delta V_R = \Delta V_{max} \sin \omega t$$

$$i_R = \frac{\Delta V_R}{R} = \frac{\Delta V_{max} \sin \omega t}{R} \qquad I_{max} = \frac{\Delta V_{max}}{R}$$

(b) $\Delta V(t) = \Delta V_{max} \sin \omega t$

$\Delta V_R = \Delta V_{max} \sin \omega t$

$i_R = \frac{\Delta V_{max}}{R} \sin \omega t$ $\Rightarrow$ there is no phase change between the current and the voltage source

(c) $P = i^2 R$

$$= \left(\frac{\Delta V_{max}}{R} \sin \omega t\right)^2 R = \frac{\Delta V_{max}^2}{R} \sin^2 \omega t$$

(d) $I_{rms} = \sqrt{\overline{i^2}}$

$i^2 = I_{max}^2 \sin^2 \omega t$

$$\overline{i^2} = \frac{1}{T}\int_0^T I_{max}^2 \sin^2 \omega t \, dt = \frac{I_{max}^2}{T}\int_0^T \sin^2 \omega t \, dt$$

$$= I_{max}^2 \frac{1}{T}\int_0^T \left[\frac{1 - \sin\frac{\omega t}{2}}{2}\right] dt$$

$$= I_{max}^2 \frac{1}{2} - \underbrace{\frac{1}{T}\int \frac{\sin\frac{\omega t}{2}}{2} dt}_{=0}$$

$$= \frac{I_{max}^2}{2}$$

$$\therefore I_{rms} = \sqrt{\overline{i^2}} = \frac{1}{2} I_{max}^2$$

2 Maxwell's equations

1. $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$ — You can calculate the electric field by constructing a Gaussian and the enclosed charge determines the field. this imply the possibility to isolate a single charge (positive or negative)

Gauss law Electricity

2. $\oint B \cdot dA = 0$ — this implies it is not possible to isolate a single pole of the magnet.

Gauss law magnetism

3 $\oint E \cdot ds = -\frac{d\phi_B}{dt}$ — the possibility to create an E field by a changing magnetic flux. this connects Electricity and magnetism

Faraday's law

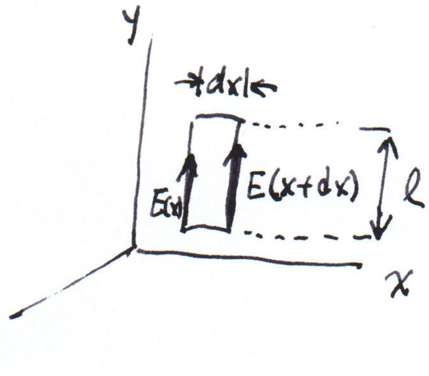
4 $\oint B \cdot ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

Ampere-Maxwell law — this determines the source of magnetis can be moving charge, or changing Electric flux ove time. and changing electric flux is a sort of current called displacement current.

3 Use Faraday's law $\oint E \cdot ds = -\dfrac{d\Phi_B}{dt}$

$\vec{E}$ field is perpendicular to $d\vec{s}$.

$$E(x+dx) \approx E(x) + \left.\frac{dE}{dx}\right|_{t=\text{const}} dx = E(x) + \frac{\partial E}{\partial x} dx$$



$$\therefore \oint \vec{E} \cdot ds = \left[E(x) + \frac{\partial E}{\partial x} dx\right] l - [E(x)] l$$

$$\simeq l \left(\frac{\partial E}{\partial x}\right) dx$$

$$\Phi_B = B l\, dx$$

$$\frac{d\Phi_B}{dt} = l\, dx \left.\frac{dB}{dt}\right|_{x=\text{Const}} = l\, dx \frac{\partial B}{\partial t}$$

From Faraday's law $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$

$$\therefore l\left(\frac{\partial E}{\partial x}\right) dx = -l\, dx \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

4 (a)

Since we are considering a plane wave, the $\vec{E}$ and $\vec{B}$

$\therefore$ fields are perpendicular to each other

$$\vec{E} \times \vec{B} = |EB| = EB$$

$$S = \frac{1}{\mu_0} \vec{E} \times \vec{B} \quad . \text{ But } \frac{E}{B} = c,$$

$$\therefore \quad S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

– Instantaneous rate of energy that is passing a unit area with instantaneous values of $\vec{B}$ an $\vec{E}$

(b) To get the time averged $\vec{S}$, we need to recall the in an EM wave, it is composed Both Electric part $[E(x,t) = E_{max} \sin(kx - \omega t)]$ and magnetic part $[B(x,t) = B_{max} \sin(kx - \omega t)]$ (or you can use $\cos(kx-\omega t)$) and we need to get the time average, ~~of~~ that is the rms of $\vec{S}$

$$\therefore I = S_{av} =$$

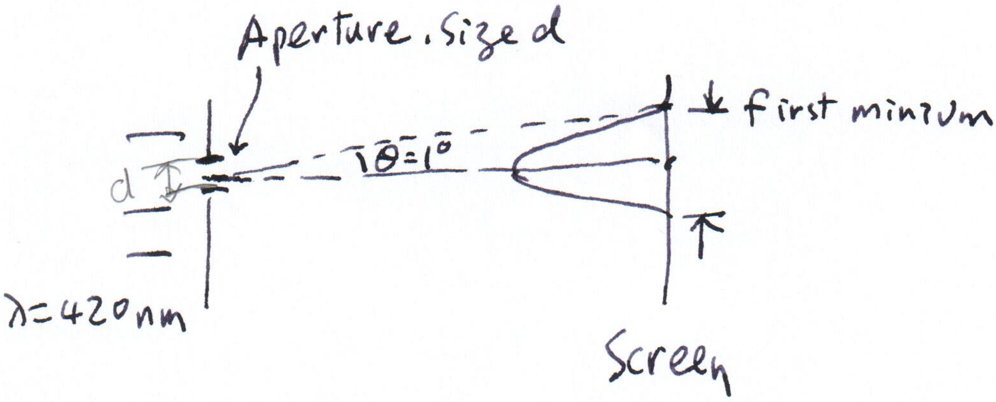
$$= S_{rms} = \frac{1}{T} \int S \, dt$$

$$= \frac{1}{T} \int_0^T \frac{1}{c\mu_0} E_{max}^2 \cos^2(kx - \omega t) = \frac{E_{max}^2}{2c\mu_0}$$

$$\text{or} = \frac{1}{T} \frac{c}{\mu_0} \int_0^T B_{max}^2 \cdot \cos^2(kx - \omega t) = \frac{c}{2\mu_0} B_{max}^2$$

#

5



The size on the screen of the aperture is the consequence of the diffraction. From the center to the first minimum is the observed size of the apeature,

the first minimum appears @

$d \sin\theta = n\lambda$, when $n=1$, and $\theta = \theta = 1°$

When angle is small, $\sin\theta \approx \theta = 2° = \frac{\pi}{180}$

To account both sides of the image

$\therefore d\left(\frac{\pi}{180}\right) = 1 \cdot 420\,nm$

$= 420 \times 10^{-9}\,m$

$\therefore d = \dfrac{420 \times 10^{-9}}{\frac{\pi}{180}} = 24\,\mu m$

or $24 \times 10^{-6}\,m$