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General Physics I, Final 1
PHYS10000AA, AB, AC, Class year 108
01-07-2020

SN:____________________, Name:_________________________

ABSOLUTELY NO CHEATING!

Note: This is a close-book examine. You can use pencil or any pen in answering the problems. Dictionary and Calculators are allowed.

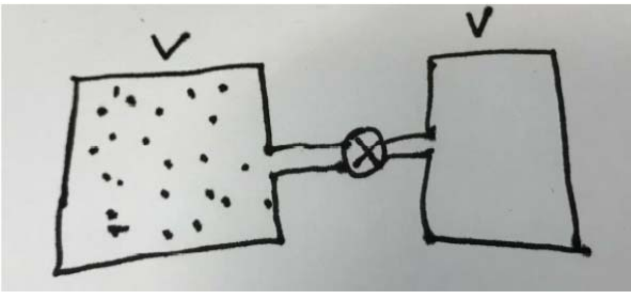
Problems (5 Problems, total 100%)

1. **Escape Speed of a Rock:** (20%) Superman picks up a rock of mass ***m*** and throws it into the space. What minimum speed must it have at the Earth's surface to move infinitely far away from the Earth?

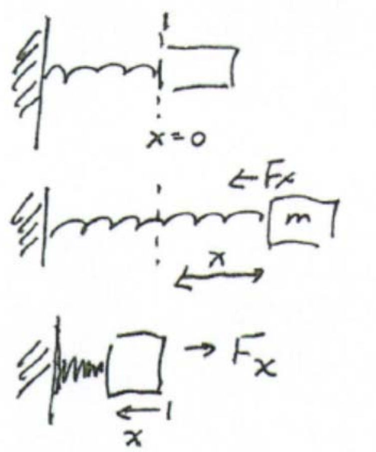
2. **Standing Wave in a string:** (20%) A taut string has a length of 2.60 m and is fixed at both ends. (a) Find the wavelength of the fundamental mode of vibration of the string. (b) Can you find the frequency of this mode? Explain why or why not.

3. **Pressure and Volume:** (20%) The deepest point in the ocean is in the Mariana Trench, about 11 km deep, in the Pacific. The pressure at this depth is huge, about 1.13×10^{8} N/m^2. (a) Calculate the change in volume of 1.00 m^3 of seawater carried from the surface to this deepest point. (b) The density of seawater at the surface is 1.03×10^{3} kg/m^3. Find its density at the bottom. (c) Explain whether or when it is a good approximation to think of water as incompressible. ***B***=Bulk module of water=0.21×10^{10} N/m^3.

4. **Free expansion:** (20%) Refer to the figure to the right. Two chambers each with a volume V are connected by a valve which can be closed or open. The left chamber has a pressure ***P***, and the right one is in vacuum. Suppose these two chambers are not connected to the outside world thermodynamically, so it can be considered as an adiabatic system. (a) If the valve is now slowly open, and after a while the system reaches thermodynamically, what is the heat change in this system? (b) How much work was done by the gas to the system? (c) What is the change of the total internal energy? In each question, explain your answer.



5. **Simple Harmonic system:** (20%) Refer to the figure to the right for a very simple mass-spring system. The attached mass is ***m***, and the spring has a spring constant ***k***. (a) Start from Newton's law, if you pull the mass with displacement ***x***, and release it; it will start to oscillate. What is its acceleration ***a***? (2) Write down the differential equation that describes the oscillation. (3) Demonstrate, without solution it, that $x(t) = Cos\ (\varpi t + \phi)$ is the solution of the equation. (4) Solve the period of the oscillation.



1. Use the concept of conservation of energy, in an earth-object system.

$$\frac{1}{2}mV_i^2 - \frac{GM_E m}{r_i} = \frac{1}{2}mV_f^2 - \frac{GM_E m}{r_f}$$

$r_i = R_E =$ radius of the earth

$r_f = r_{max}$; $V_f = 0$ when it reach the r_{max}

$$\therefore \frac{1}{2}mV_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}}, \quad r_{max} \to \infty$$

if the rock is to escape from the earth

$$\Rightarrow \frac{1}{2}mV_i^2 = \frac{GM_E m}{R_E}$$

$$V_i = V_{escape} = \sqrt{\frac{2GM_E}{R_E}}$$

2. The string is taut and fixed on it both ends, therefor Standing wave can form, and ~~th~~ have nodes $\lambda/2$ apart

(a) $\Rightarrow \lambda_n = \frac{2L}{n}$, $n =$ integers $= 1, 2, 3 \ldots$

Fundamental means $n = 1$. $\lambda_1 = 2L$

$= 2 \times 2.60$

$= 5.20m$

(b) No, we don't know the speed of this wave, therefore we can't find the frequency.

the frequency $f_n = n\frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

To know the frequency, we need either v (the speed) or T, μ ($T =$ period, $\mu =$ string density)

3. Use $B = -\frac{\Delta P}{\Delta V / V_i} = -\frac{\Delta P V_i}{\Delta V}$ B = Bulk module of water $= 0.21 \times 10^{10} \frac{N}{m^2}$

(a) ΔV = volume change

$$= -\frac{\Delta P V_i}{\Delta V} = \frac{(1.13 \times 10^{8} \frac{N}{m^2})(1 m^3)}{0.21 \times 10^{10} N/m^2} = \cancel{-0.05} = -0.0538 m^3$$

(b) at the surface the density of water is $1.03 \times 10^3 \, kg/m^3$
and the mass of $1 m^3$ is 1.03×10^3 kg
at the bottom the mass ~~is~~ occupied the volume is

$$V_{bottom} = 1 m^3 - 0.0538 m^3 = 0.946 m^3$$

$$\therefore D_{at\ bottom} = \frac{1.03 \times 10^3 kg}{0.946 m^3} = 1.09 \times 10^3 \, kg/m^3$$

(c) ΔV = volume change from the surface to the bottom $= -0.0538 m^3$

$\simeq 5\%$

Therefore, water as a liquid is indeed incompressible, even at the etreme case.

4. for a free expansion. and it is adiabatic

$$\Delta E_{int} = \Delta Q - \Delta W$$

in this system. No heat exchange, so

(a) $\Delta Q = 0$. no heat exchanged

(b) It is vacuum on the other chamber, so $\Delta W = 0$

(c) From 1st law of thermodynamic. Sin $\Delta Q = \Delta W = 0$

$\therefore \Delta E_{int} = 0$. therer is no internal energy change

5

(a) $F = -kx = m a_x$

$\therefore a_x = -\frac{k}{m} x$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{dx^2}{dt^2} = -\frac{k}{m} x$

$\therefore \frac{d^2x(t)}{dt^2} + \frac{k}{m} x(t) = 0$ 2nd order differential equation

(c) If $x(t) = A\cos(\omega t + \phi)$ is its soution. we can plug in this to the differential equation and it will satisfy this eqiation

Plugin

$$\frac{d^2x(t)}{dt^2} = \frac{d}{dt}\left[\frac{d}{dt}x(t)\right] = \frac{d}{dt}\left[\frac{d}{dt}A\cos(\omega t+\phi)\right]$$

$$= \frac{d}{dt}\left[-A\omega\sin(\omega t+\phi)\right]$$

$$= -A\omega^2\cos(\omega t+\phi)$$

$$= -\omega^2 x(t)$$

$$\therefore \frac{d^2x(t)}{dt^2} + \omega^2 x(t) = 0$$

from (b) $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ $\Rightarrow \omega^2 = \sqrt{\frac{k}{m}}$ = frequency

(d) for this oscillation $x(t) = A\cos(\omega t+\phi)$

$x(t) = x(t+T)$

$\therefore [\omega(t+T)+\phi] - (\omega t+\phi) = 2\pi$

$\therefore \omega T = 2\pi$, or $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$