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Chapter 11

1. A uniform solid sphere of radius $r = 0.500$ m and mass $m = 15.0$ kg turns counter clockwise about a vertical axis through its centre. Find its vector angular momentum about this axis when its angular speed is 3.00 rad/s.

Ans:

The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0 \text{ kg})(0.500 \text{ m})^2 = 1.50 \text{ kg} \cdot \text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = (1.50 \text{ kg} \cdot \text{m}^2)(3.00 \text{ rad/s}) = 4.50 \text{ kg} \cdot \text{m}^2/\text{s}$$

Since the sphere rotates counter clockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus,

$$\vec{L} = (4.50 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\mathbf{k}}$$

due to the gravitational force on the ball.

2. The angular momentum vector of a precessing gyroscope sweeps out a cone as shown in Figure P11.31. The angular speed of the tip of the angular momentum vector, called its precessional frequency, is given by $\omega_p = \tau/L$, where τ is the magnitude of the torque on the gyroscope and L is the magnitude of its angular momentum. In the motion called *precession of the equinoxes*, the Earth's axis of rotation precesses about the perpendicular to its

orbital plane with a period of 2.58×10^4 yr. Model the Earth as a uniform sphere and calculate the torque on the Earth that is causing this precession.

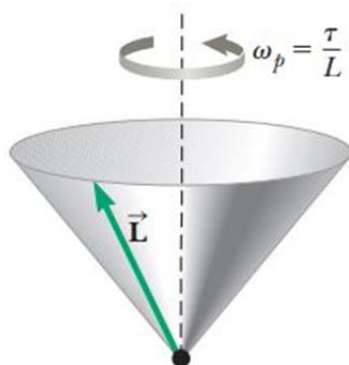


Figure P11.31 A precessing angular momentum vector sweeps out a cone in space.

Ans:

We begin by calculating the moment of inertia of the Earth, modelled as a sphere:

$$I = \frac{2}{5} MR^2 = \frac{2}{5} (5.98 \times 10^{24} \text{ kg}) (6.37 \times 10^6 \text{ m})^2$$

$$= 9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2$$

Earth's rotational angular momentum is then

$$L = I\omega = (9.71 \times 10^{37} \text{ kg} \cdot \text{m}^2) \left(\frac{2\pi \text{ rad}}{86400 \text{ s}} \right) = 7.06 \times 10^{33} \text{ kg} \cdot \text{m}^2 / \text{s}^2$$

from which we can calculate the torque that is causing the precession:

$$\tau = L\omega_p$$

$$= (7.06 \times 10^{33} \text{ kg} \cdot \text{m}^2 / \text{s}) \left(\frac{2\pi \text{ rad}}{2.58 \times 10^4 \text{ yr}} \right) \left(\frac{1 \text{ yr}}{365.25 \text{ d}} \right) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right)$$

$$= \boxed{5.45 \times 10^{22} \text{ N} \cdot \text{m}}$$