Chapter 7.

Energy and energy transfer
Energy of a system

- A new approach in solving the mechanic motion, without using Newton's laws.
- An important concept : conservation of energy.
- Identifying "system".

In this system, All object obey "energy conservation"

Work :

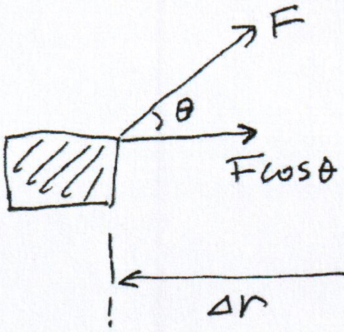


Fig 7-2.

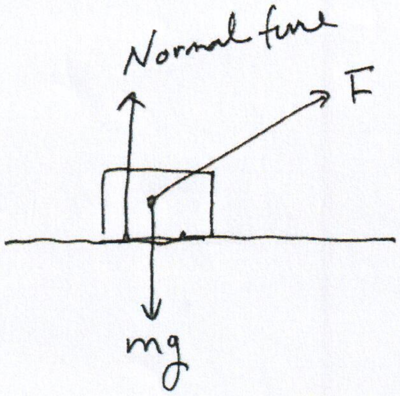
How effective the force is moving the object?

① the ~~magnitude~~ magnitude of the force

② The direction of the force

$W \equiv F \Delta r \cos\theta$ definition of work.

the work done not only depends the magnitude of the force but also the angle and the displacement of the work.



From the definition

① The gravitation force doesn't do work

② The Normal force does not do work

$$W \equiv F \Delta r \cos\theta$$
$$= \vec{F} \cdot \vec{r}$$

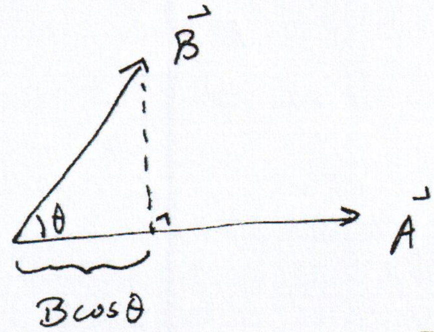
The sign of work depends on the direction of $\vec{r}$

"+": the same as the force

"−", oppositive to the Force.

<u>Mathematics</u> scalar product.

$\vec{A} \cdot \vec{B} = AB\cos\theta$. $\theta \equiv$ the angle between $\vec{A}$ and $\vec{B}$



① The projection of $\vec{B}$ along the $\vec{A}$ direction

② The scalar product $\vec{A} \cdot \vec{B}$ is <u>NOT</u> a vector

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$
$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \cancel{\vec{B}}\,\vec{A} \cdot \vec{C}$$

Unit vectors $\hat{i}, \hat{j}, \hat{k}$ in a righ-handed coordinate system.

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$
$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \quad \text{— orthogonal}$$

Any vector in this coordinate system can be decomposed into 3 components along the three unit vector system.

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_k \hat{k}$$
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_K \hat{k}$$
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$\vec{A} \cdot \vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2.$$

Work done by a varying force

$$W = F_x \Delta x$$

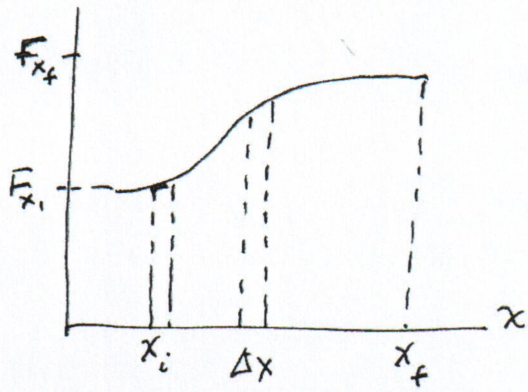
$$\approx \sum_{x_i}^{x_f} F_x \Delta x$$

$$= F_{x_1} \Delta x_1 + F_{x_2} \Delta x_2$$

$$= \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x$$

$$= \int_{x_i}^{x_f} F_x \, dx$$

$$= W = \text{the area under the curve.}$$



for more than one force acting on the system (or, particle)

$$\sum W = W_{total} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

Example: work done by a spring-block system

$$F_x = -kx$$

↑ spring constant

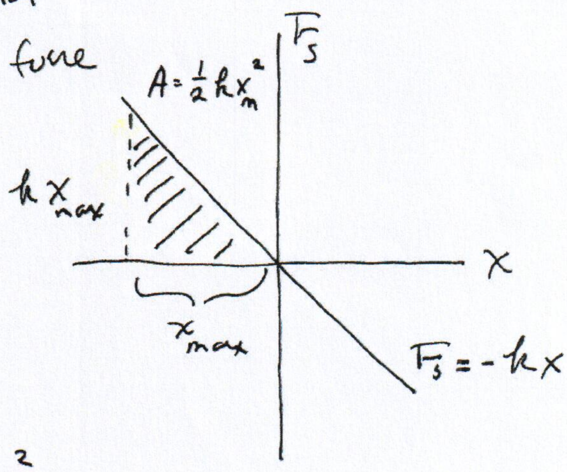
Slope of the force

$$W_s = \int_{x_i}^{x_f} F_s \, dx$$

$$= \int_{-x_{max}}^{0} (-kx) \, dx$$

$$= \left[\frac{-1}{2} k x^2\right]_{-x_{max}}^{0} = \frac{1}{2} k x_{max}^2$$

maximum work that can be done by the force at x_{max} displacement.



Suppose the force is pulling to the $+\hat{x}$ direction

$\vec{F} = -kx$ (means the force of the spring in $-\hat{x}$ direction

$$W_s = \int_0^{x_{max}} (-kx)\,dx = \left[\frac{-1}{2}kx^2\right]_0^{x_{max}}$$

$$= -\frac{1}{2}kx_{max}^2$$

Suppose the force is pulling to the $+\hat{x}$ direction, $F_{app} = kx$

$F_{sp}(x) = -kx$ ⇒ the force of the spring is in the $-\hat{x}$ direction

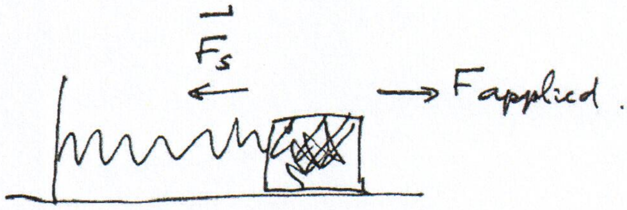
the work done by the $\vec{F}$ is

$$W_{app} = -W_s = -\int_0^{x_{max}} (-kx)\,dx = \left[\frac{1}{2}kx\right]_0^{x_{max}}$$

$$= \frac{1}{2}kx_{max}^2$$

This means this much of energy will be stored in the spring as elastic potential energy in general

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$



$$W_{app} = \int_0^{x_{max}} F_{app} \cdot dx$$

$$= \int_0^{x_{max}} kx\,dx = \frac{1}{2}kx_{max}^2$$

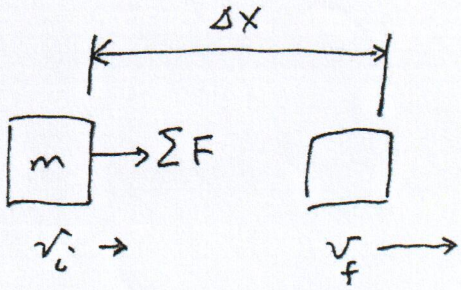
$$W_{app} = \int_{x_i}^{x_f} F_{app}\,dx = \int_{x_i}^{x_f} kx\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Note $\boxed{W_{app} = -W_s}$

work as the mechanism of transfering energy into a system

- : Change the speed.
- : Change the temperature
- : ...

Kinetic energy



$$\Sigma W = \int_{x_i}^{x_f} \Sigma F\,dx$$

$$= \int_{x_i}^{x_f} ma\,dx$$

$$= \int_{x_i}^{x_f} m\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\frac{dv}{dx}\frac{dx}{dt}\,dx$$

$$= \int_{v_i}^{v_f} mv\,dv$$

$$= \underbrace{\frac{1}{2}mv_f^2}_{E_{Kf}} - \underbrace{\frac{1}{2}mv_i^2}_{E_{Ki}}$$

— Work done by force, ΣF, results in a change in the velocity that change the energy

→ kinetic energy

$K \equiv \frac{1}{2}mv^2$

$\therefore \quad \sum W = K_f - K_i = \Delta K$

— Work-Kinetic energy theorem

→ Non isolated system : the system interact with its environment

friction

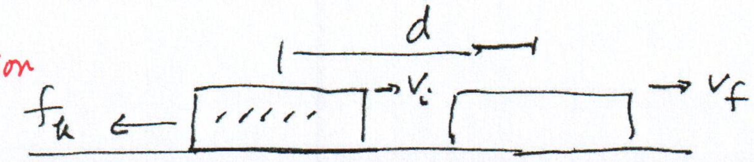


Fig 7-16

The book slows down (Kinetic energy decreases) the rest of the energy transferred to the surface as heat (internal energy)

$$(\sum F_x)\Delta x = (ma_x)\Delta x$$ — Newton's 2nd law

$$a_x = \frac{v_f - v_i}{t}, \quad \Delta x = \frac{1}{2}(v_i + v_f)t$$

$v = v_0 + at$

— A particle under constant acceleration.

$$\therefore \quad (\sum F_x)\Delta x = m\left(\frac{v_f - v_i}{t}\right)\frac{1}{2}(v_i + v_f)t$$

$$(\sum F_x)\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

↑ the displacement of the large object (not the point)

$$= (-f_k)\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \Delta K$$

$$\therefore \quad -f_k d = \Delta K$$

$$\therefore \quad \Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$\Delta E_{system} = \underset{\text{Energy transferred}}{\sum T} = \Delta K + \Delta E_{int} = 0$$ Conservation of energy

$$-f_k d + \Delta E_{int} = 0 \ . \rightarrow \ \Delta E_{in} = f_k d$$

The result of a friction force is to transform Kinetic energy into internal energy, and the increase in internal energy is equal to the decrease in Kinetic energy.

Power

$$\bar{P} \equiv \frac{W}{\Delta t} = \lim_{\Delta t \to 0} \frac{W}{dt} = \frac{dW}{dt}$$

$$= \vec{F} \cdot \frac{d\vec{r}}{dt}$$

$$= \vec{F} \cdot \vec{v}$$

Energy is not a vector

$$\equiv \frac{dE}{dt}$$ energy transfer

$$= \frac{dE}{dt}$$

$1 W = 1 J/sec = 1 kg \cdot m^2/s^3$

$1 hp = 746 W$

hp = horse power