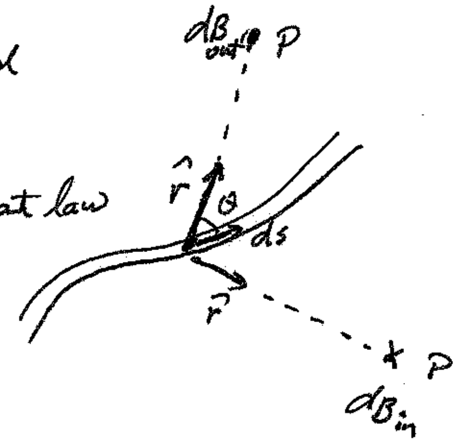
Chap 30. Source of the magnetic field

30.1 Biot-Savart law

Current produces magnetic field



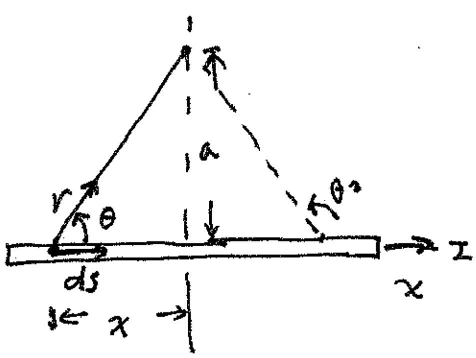
$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \quad \text{— Biot Savart law}$$

1) $dB \perp ds$ and $dB \perp \hat{r}$
2) $dB \propto \frac{1}{r^2}$
3) $dB \propto I$
4) $dB \propto \sin\theta$ $\qquad \mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$
$\qquad\qquad$ = permeability of free space

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

example: Magnetic field of a thin straight current



$$d\vec{s} \times \hat{r} = |d\vec{s} \times \hat{r}|\, \hat{k}$$
$$= (ds \sin\theta)\, \hat{k}$$
$$= (dx \sin\theta)\, \hat{k}$$

$$d\vec{B} = (dB)\, \hat{k} = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2} \hat{k}$$

$$= \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2}$$

$$r = \frac{a}{\sin\theta} = a \csc\theta$$

$$x = -a \cot\theta$$

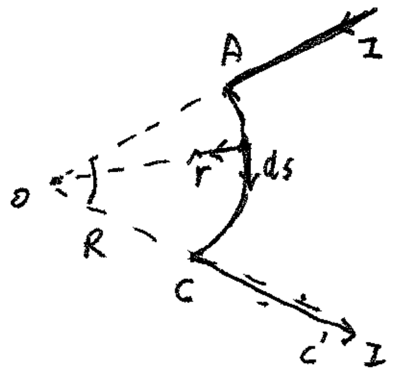
$$dx = a \csc^2\theta$$

$$\therefore dB = \frac{\mu_0 I}{4\pi} \frac{a \csc^2\theta \sin\theta\, d\theta}{a^2 \csc^2\theta} = \frac{\mu_0 I}{4\pi a} \sin\theta\, d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2) \qquad \begin{aligned} \theta_1 &= 0 \\ \theta_2 &= 180^\circ \end{aligned}$$

$$B = \frac{\mu_0 I}{2\pi a}$$

2) Magnetic field due to curved Wire Segment



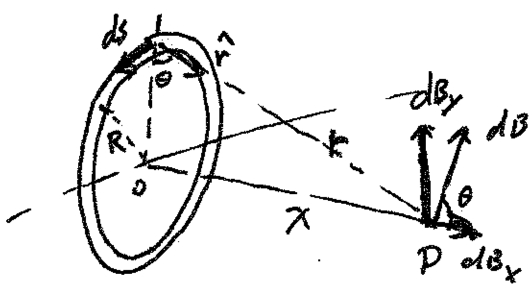
$$dB = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}$$

$$B = \int dB = \frac{\mu_0 I}{4\pi R^2}\int_0^s ds = \frac{\mu_0 I}{4\pi}\frac{s}{R^2}$$

But $s = R\theta$

$$\therefore B = \frac{\mu_0 I}{4\pi}\frac{\theta}{R}$$

3) Circular Current loop



$$r^2 = x^2 + R^2$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s}\times\hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{(x^2+R^2)}$$

$$dB_x = dB\cos\theta \rightarrow B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint\frac{ds\cos\theta}{x^2+R^2}$$

$$\cos\theta = \frac{R}{(x^2+R^2)^{1/2}}$$

$$\therefore B_x = \frac{\mu_0 I}{4\pi}\frac{1}{(x^2+R^2)^{3/2}}\oint ds \qquad \oint ds = 2\pi R$$

$$\therefore B_x = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$

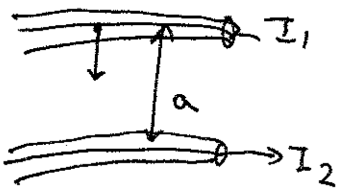
$B_y = 0$ symmetric

① at $x = 0$

$$B_x = \frac{\mu_0 I}{2R}$$

② If $x \gg R$, $\rightarrow B_x = \dfrac{\mu_0 I R^2}{2x^3}$

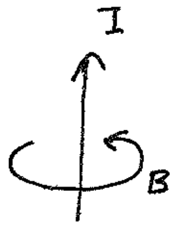
30.2 Magnetic force between two parallel conductors



$$F_1 = I_1 l B_2$$

$$= I_1 l_1 \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0}{2\pi} I_1 I_2 l$$

30.3 Ampere's law

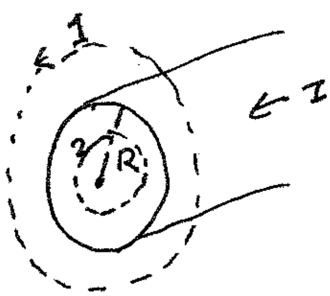


$$\oint B \cdot ds = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

$$\therefore \oint B \cdot ds = \mu_0 I$$

$I \equiv$ current enclosed

Ex: Long current-carrying wire



① $r >> R$

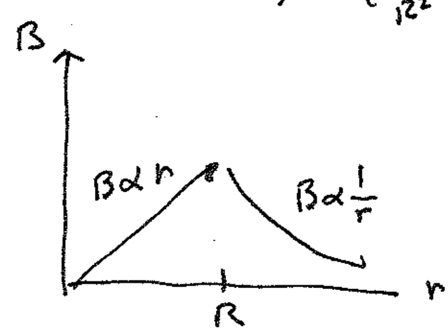
$$\oint B \cdot ds = B \oint ds = B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{for } r >> R$$

② $r < R$ $\quad \oint B \cdot ds = B(2\pi r) = \mu_0 I'$, $\quad I' = \left(\frac{\pi r^2}{\pi R^2}\right) I$

$$\therefore B = \frac{1}{2\pi r} \mu_0 \frac{\pi r^2}{\pi R^2} I$$

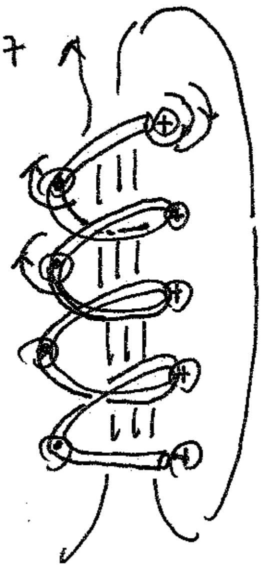
$$= \mu_0 \left(\frac{r^2}{R^2}\right) I = \frac{\mu_0 I}{2\pi R^2} r$$



30.4 Magnetic field of a Solenoid

Solenoid = long wire in a form of a helix

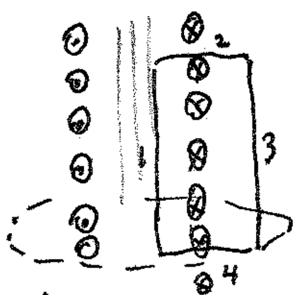
Fig 30.17



Uniform magnetic field can be obtained

Ideal Solenoid

→ the turns are closely packed and the length is much greater than the radius



Use this Ampere loop to calculate the field outside the loop

— closely packed

$\vec{B}$ is inside and parallel

Use Ampere's law.

the field outside the Solenoid is zero

Only path 1 has contribution

$$\oint B \cdot ds = \int_{\text{path 1}} B \cdot ds = B \int ds = B\ell = \mu_0 I$$

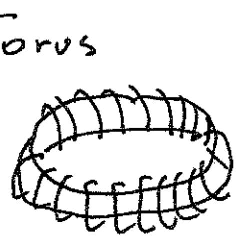
for N turns

$$\oint B \cdot d\ell = B\ell = \mu_0 N I$$

$$\therefore \boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I}$$

$n = \frac{N}{\ell}$ = number of turns per unit length.

Torus



N turns

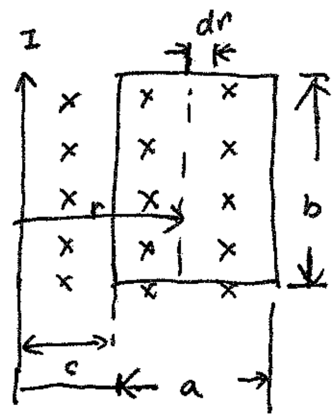
$n = \frac{N}{2\pi r} \Rightarrow \quad B = \mu_0 n I = \mu_0 \frac{N}{2\pi r} I$ eq.(30.16)

30.5 Magnetic flux

Similar to electric flux

$\Phi_B \equiv \int B \cdot dA$. for a plane of area A . in a uniform B

$$\Phi_B = BA\cos\theta$$



$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int \frac{\mu_0 I}{2\pi r} dA$$

$$= \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$

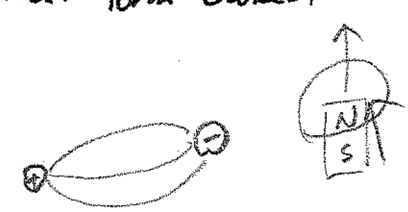
30.6 Gauss law in magnetism

$$\oint B \cdot dA = 0$$

Note: $\epsilon_0 \oint E \cdot dA = q_{in}$

because magnetic field ~~is~~ lines form closed loop. No begining no ends

Check Fig 30.23 Fig 30.24



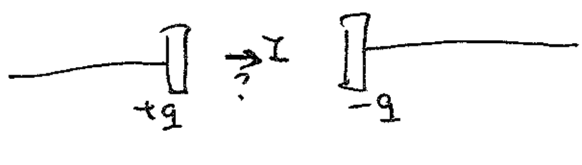
30.7 Displacement current and General form of Ampere's law

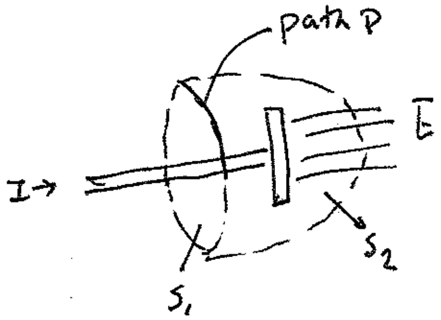
Change in motion → produce magnetic field.

$\oint B \cdot ds = \mu_0 I$, $I = \frac{dq}{dt}$

→ Only valid when electric field present is constant in time

But in a charging capacitor, when charging. There is no conduction current in the circuit.





1) Path P as boundary of S_1
Real current passes it

2) path P as boundary of S_2.
No conduction current passes it.

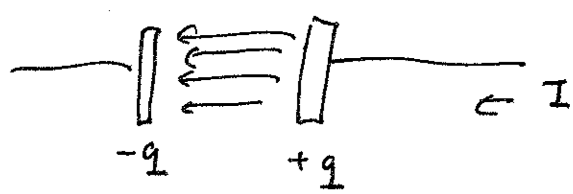
→ Contradictory situation.
due to the discontinuity of current.

Maxwell postulated an additional term in the Ampere's law called displacement current

$$I_d \equiv \text{displacement current} \cong \epsilon_0 \frac{d\Phi_E}{dt} \quad . \quad \Phi_E = \int E \cdot dA$$

$$\therefore \oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

in Fig 30.26

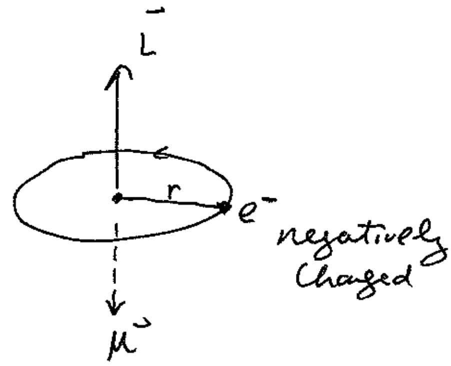


$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} = I$$ Same as the conduction current I.

30.8 Magnetism in Matter

Consider an electron moving in an atom.



$$I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \frac{1}{2}evr$$

But L = orbital angular momentum $= m_e v r$

$$\therefore \mu = \frac{m}{2m} evr = \left(\frac{e}{2m}\right)L$$

→ the magnetic momentum is proportional to its orbital angular momentum
But since electron is negatively charged - so ~~it is~~ the directions of $\vec{\mu}$ and $\vec{L}$ are opposite to each other.

$$\mu = \frac{e}{2m}\vec{L} \quad = \text{Classically}$$

But Quantum Mechanically, $\vec{L} = \sqrt{l(l+1)}\,\hbar$

Quantized to $\hbar$

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\ \text{J}\cdot\text{sec}$$

$\therefore$ the smallest $L = \sqrt{2}\,\hbar$

Thus $\mu_{orb} = \frac{e}{2m}\sqrt{2}\,\hbar$

→ The magnetic effect produced by the orbital motion of the electrons

Another Characteristic of electron → Spin $\equiv S$

S = Spin angular momentum $= \sqrt{s(s+1)}\,\hbar$

s can take only $\pm s$

Smallest $s = \frac{1}{2}$. $S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$

$$\mu_s = -\frac{g_s \mu_B}{\hbar} s = -g\frac{e\hbar}{\hbar\, 2m}\cdot\frac{1}{2}$$

$$\mu_{spin} = \frac{e\hbar}{2m}$$

$\mu_{spin} =$ ~~[illegible]~~ $\vec{s}$ $\quad \mu_B = \frac{e\hbar}{2m} = 9.27 \times 10^{-24}\ \text{J/T}$ = Bohr magneton

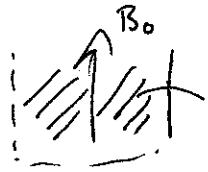
$=$ ~~[illegible]~~ $\vec{s}$

But $g = 2$ for spin

Magnetization Vector and Magnetic field Strength

magnetization = The magnetic State of a Substance
→ Magnetization Vector $\vec{M}$

magnetic moment per unit volume of substance



filled with magnetic substance

Then $B = B_0 + B_m$

B_0 = magnetic field produced by current carrying conductor

B_M = the field produced by the magnetic substance

Suppose B_M is produced by small Solenoids

$$B_m = \mu_0 n I = \mu_0 \frac{N}{l} I = \mu_0 \frac{NIA}{lA}$$ — total magnetic moments

$$= \mu_0 \frac{\mu}{V} \rightarrow \text{define } M \equiv \frac{\mu}{V} \quad V = \text{volume}$$

$\therefore \; B = B_0 + \mu_0 M$ define magnetic strength $H = \dfrac{B_0}{\mu_0}$

$\therefore \; B = \mu_0 (H + M)$

M produced by magnetic Substances.

Classification of Magnetic Substance

Paramagnetic , ferromagnetic , Diamagnetic

① paramagnetic $M = \chi H$. M is proportional to H

$\chi \equiv$ magnetic susceptibility = positive

② Diamagnetic χ is negative

$$B = \mu_0 (H + M) = \mu_0 (H + \chi H) = \mu_0 (1 + \chi) H$$

$$= \mu_m H$$

$\mu_m = \mu_0 (1 + \chi)$. para : $\mu_m > \mu_0$

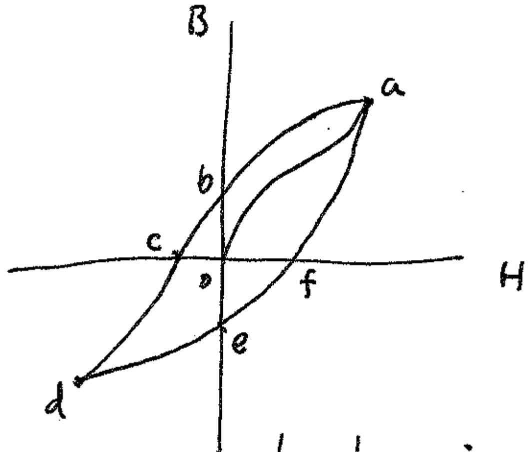
diamag : $\mu_m < \mu_0$

③ Ferromagnetism

— A small number of crystalline substances exhibit strong magnetic effect called Ferromagnetism → ion, Cobalt nickel

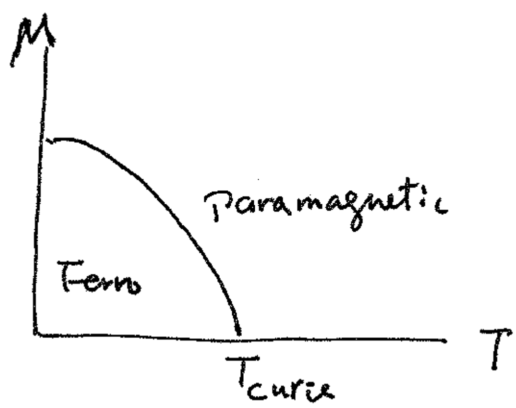
→ Contain permanent atomic magnetic moment that tend to align parallel to each other even a welk external magnetic field.

→ The microscopic regions called <u>domains</u>.



H = applied magnetic field
B = total magnetic field.

hysteresis curve



Paramagnetism $0 < \chi \ll 1$ $\quad M = c\,\frac{B_0}{T}$ Curie's law

Pierre Curie