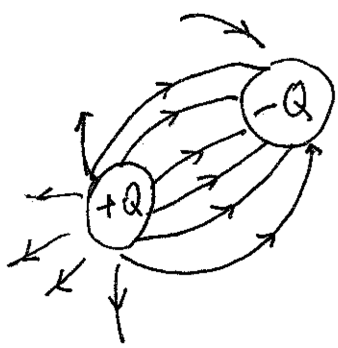
Chap26 Capacitance and Dielectrics

26.1 Definition



There is a potential difference due to the charge on each plates

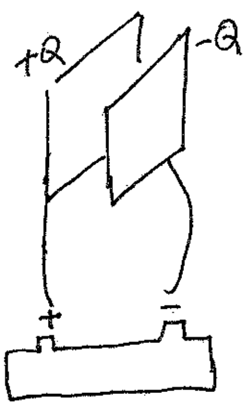
$Q \propto \Delta V$

$Q = C\Delta V$. C = proportional constant

$\therefore C \equiv \dfrac{Q}{\Delta V}$ $\quad 1[F] = 1 \dfrac{\text{Coulomb}}{\text{Volt}}$

= Capacitance, always a positive number.

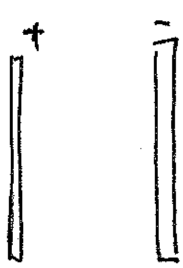
= capacitor's ability to hold the charge



$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e \frac{Q}{R}} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

for a spherically charge sphere.

Parallel-plate capacitor



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed \quad (25.6)$$

$$= \frac{Qd}{\epsilon_0 A}$$

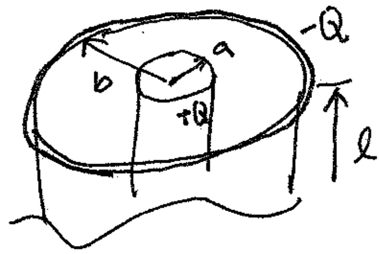
$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{Qd}{\epsilon_0 A}}$$

$$\therefore C = \frac{\epsilon_0 A}{d}$$

$C \sim A$ (large)

$C \sim \frac{1}{d}$ (Short and thin)

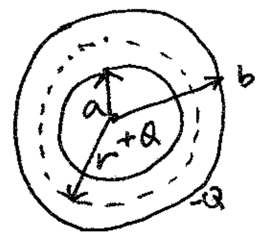
Cylindirical Capacitor



$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$$

Using Gauss's law

$$E = 2k_e \frac{\lambda}{r} \quad (24.7),$$



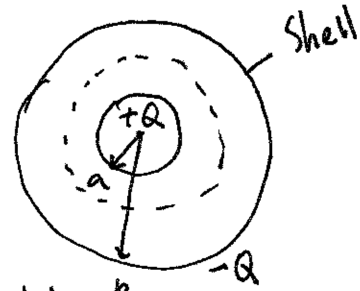
$\lambda = \frac{Q}{l}$

$$\therefore V_b - V_a = -\int_a^b E_r dr = -2k_e \lambda \int_a^b \frac{dr}{r}$$

$$= -2k_e \lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(2k_e \frac{Q}{l}\right) \ln\left(\frac{b}{a}\right)} = \frac{l}{2k_e \ln\left(\frac{b}{a}\right)}$$

Spherical Capacitor



$$V_b - V_a = -\int_a^b E_r dr =$$

$$= -k_e Q \int_a^b \frac{dr}{r^2}$$

$$= k_e Q \left[\frac{1}{r}\right]_a^b = k_e Q \left(\frac{1}{b} - \frac{1}{a}\right)$$

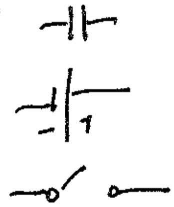
$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q \left(\frac{1}{b} - \frac{1}{a}\right)} = \frac{ab}{k_e (b-a)}$$

26.3 Combination of Capacitors

Capacitor

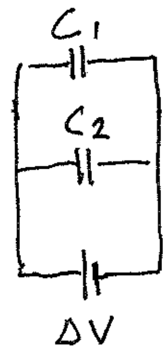
Battery

Switch



circuit

① Parallel Combination



the individual potential across each capacitor is the same

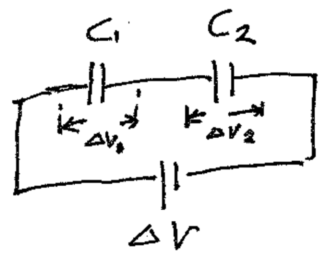
$Q = Q_1 + Q_2$

$Q_1 = C_1 \Delta V \quad Q_2 = C_2 \Delta V$

$$
\begin{aligned}
Q &= C_1 \Delta V + C_2 \Delta V \\
&= (C_1 + C_2)\Delta V \\
&= C_{eq}
\end{aligned}
$$

$\therefore \rightarrow \boxed{C_{eq} = C_1 + C_2}$

② Series Combination



$\Delta V = \Delta V_1 + \Delta V_2$

$\Delta V_1 = \dfrac{Q}{C_1} \quad \Delta V_2 = \dfrac{Q}{C_2}$

$$\Delta V = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q$$

$\rightarrow \boxed{Q_{eq} = \dfrac{1}{C_1} + \dfrac{1}{C_2}}$

→ the change on each capacitor is the same for each capacitor.

26.4 Energy Stored in Charged Capacitor

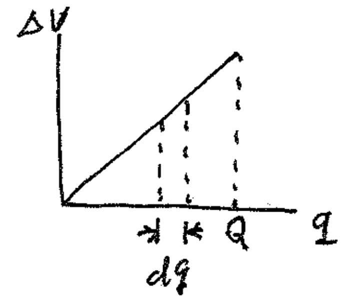
the work needed to transfer charge q in a capacitor

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{2C} Q^2$$

the work is the energy that can store in the capacitor

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2} C V^2 \qquad (Q = C\Delta V)$$



for a parallel plate capacitor

$$\Delta V = E d$$

$$C = \frac{\epsilon_0 A}{d} \quad (26.3)$$

$$\therefore\ U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2$$

$$= \frac{1}{2}\epsilon_0 A d\, E^2$$

Define energy density $\equiv u =$ energy per unit volume

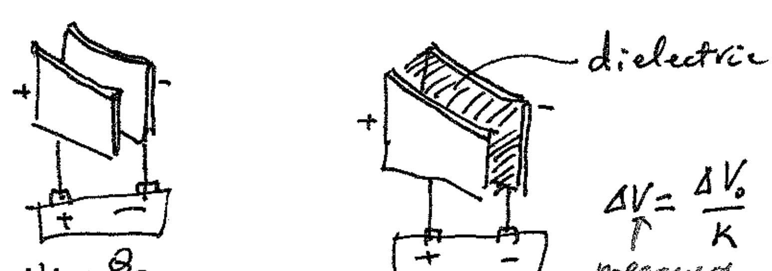
$$= \frac{U}{Ad}$$

$$\therefore\ u = \frac{1}{2}\epsilon_0 A d\, E^2 \frac{1}{Ad} = \frac{1}{2}\epsilon_0 E^2$$

$$\therefore \boxed{u = \frac{1}{2}\epsilon_0 E^2}$$ energy density of the capacitor

26.5 Capacitor with dielectric

dielectric : Non conducting material.
when a capacitor is filled with a dielectric material, its capacitance increases by a factor of K (called dielectric constant).



$$\Delta V_0 = \frac{Q_0}{C}$$

$$\Delta V = \frac{\Delta V_0}{K}$$

measured

$\Delta V = \frac{\Delta V_0}{K}$, $\Delta V < \Delta V_0$, $\underline{\underline{K > 1}}$ positive.

$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / K} = K \frac{Q_0}{\Delta V_0}$ → The New Capacitor increases by a factor a K due to the filling of dielectric

$\therefore C = K C_0$

But $C_0 = \epsilon_0 \frac{A}{d}$ (26.3)

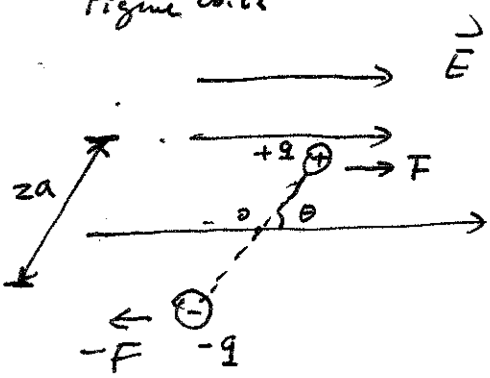
$\therefore \boxed{C = K \frac{\epsilon_0 A}{d}}$

dielectric:

1) Increase in Capacitor
2) Increase in maximum operating voltage
3) Mechanic Support between the plates of the Capacitor.

26.6 Electric dipole in an electric field.

Figure 26.22



electric dipole moment

$P \equiv 2qa$ (Note: define the distance differently)

$F = qE$ net force is zero

the two forces produce a net torque on this dipole

$\tau = 2Fa \sin\theta$ $F = qE$, $P = 2qa$

$= 2qEa \sin\theta$

$= PE \sin\theta$

$\Rightarrow \boxed{\vec{\tau} = \vec{P} \times \vec{E}}$

The potential of this system: A dipole in an electric field.
→ The work done to rotate the dipole will stored as potential energy in this system

$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau \, d\theta = \int_{\theta_i}^{\theta_f} PE \sin\theta \, d\theta = PE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$

$= PE(\cos\theta_i - \cos\theta_f)$

It is convenient for us to choose $\theta_i = 90^\circ$, $U_i = 0$

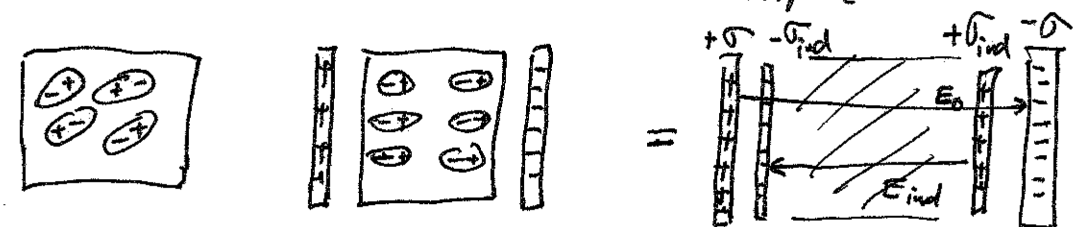
$$\therefore \boxed{U_f = -pE\cos\theta} = -\vec{p}\cdot\vec{E}$$

26.7 An atomic description of Dielectric.

When a dielectric is inserted $\Delta V = \frac{\Delta V_0}{K} \rightarrow E = \frac{E_0}{K}$

1) If dielectrics are polar material, the electric field causes an alignment of the dipoles.

2) If dielectric are nonpolar → electric field causes an induced dipole



1) Randomly oriented

2) Electric field is applied

The charged edge of the dielectric can be modeled as additional pair of parallel plates

∴ After inserting the dielectric

$E = E_0 - E_{ind}$, E = electric field in the dielectric

$$\frac{\sigma}{K\epsilon_0} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_{ind}}{\epsilon_0} \Rightarrow \sigma_{ind} = \left(\frac{K-1}{K}\right)\sigma$$

Since $K > 1$

$\therefore \sigma_{ind} < \sigma$

Note: If the dielectric is replaced by a conductor $(E = 0)$

$$E_0 = E_{ind} \rightarrow \sigma_{ind} = \sigma$$

Do Example 26.9 page 819.