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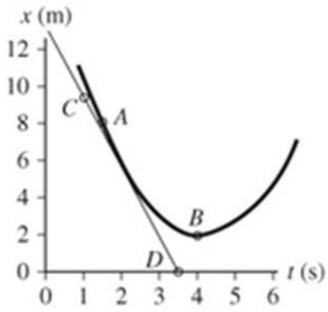
General Physics I, Midterm 1s
PHYS10000AA, AB, AC, Class year 107
10-25-2017

Solutions
SN:___________________, Name:________________

Note: You can use pencil or any pen in answering the problems. Dictionary, calculators and mathematics tables ***are*** *allowed. Please hand in both solution and this problem sheet.* ***ABSOLUTELY NO CHEATING!***

Problems (total 6 problems, 120%)

1. **Position, Time and velocity: (20%)** A position-time graph for a particle moving along the x axis is shown in the figure to the right. (a) Find the average velocity in the time interval $t = 1.50$ s to $t = 4.00$ s. (b) Determine the instantaneous velocity at $t = 2.00$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?



Ans: For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.
We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\text{ m}}{(4.0 - 1.5)\text{ s}}$$
$$= -\frac{6.0\text{ m}}{2.5\text{ s}} = \boxed{-2.4\text{ m/s}}$$

(b) The slope of the tangent line can be found from points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8\text{ m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative x direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where x has its minimum value. This is at $t \approx \boxed{4.0\text{ s}}$.

2. **Kinematics: (20%)** The front 1.20 m of a 1 400-kg car is designed as a "crumple zone" that collapses to absorb the shock of a collision. If a car traveling 25.0 m/s stops uniformly in 1.20 m, (a) how long does the collision last, (b) what is the magnitude of the average force on the car, and (c) what is the acceleration of the car? Express the acceleration as a multiple of the acceleration due to gravity.

(a) From the kinematic equations,

$$\Delta t = \frac{\Delta x}{\upsilon_{\text{avg}}} = \frac{2\Delta x}{\upsilon_f + \upsilon_i} = \frac{2(1.20\,\text{m})}{0 + 25.0\,\text{m/s}} = \boxed{9.60\times10^{-2}\,\text{s}}$$

(a) We find the average force from the momentum-impulse theorem:

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta\upsilon}{\Delta t} = \frac{(1\,400\,\text{kg})(25.0\,\text{m/s} - 0)}{9.60\times10^{-2}\,\text{s}} = \boxed{3.65\times10^{5}\,\text{N}}$$

(b) Using the particle under constant acceleration model,

$$a_{\text{avg}} = \frac{\Delta\upsilon}{\Delta t}\frac{25.0\,\text{m/s} - 0}{9.60\times10^{-2}\,\text{s}} = (260\,\text{m/s}^2)\left(\frac{1g}{9.80\,\text{m/s}^2}\right) = \boxed{26.5g}$$

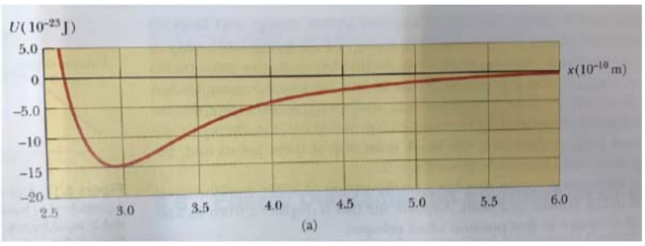
3. **<u>Lennard-Jones Potential</u>: (25%)** The potential energy associated with the force between two neutral atoms in a molecule can be modeled by the Lennard-Jones potential as $U(x) = 4\varepsilon\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6}\right]$, where **x** is the separation of the atoms. (a)What is the most likely distance between the two atoms? (10%) (b) Given σ=0.263 nm, and $\varepsilon=1.51\times10^{-22}$J are two typical constants in a molecule, what is the atom separation in a typical chemical bond? (5%) (c) Draw the potential curve qualitatively (5%) (d)When the two atoms are separated at a distance of 4.5×10^{-10} m, the two atoms are subject to a restoration or repelling force? (5%), (e) Explain you answer in (d) (5%)

(a) The separation of two atoms is where the potential is in its minimum. To find the minimum, we set $\frac{dU(x)}{dx} = 4\varepsilon\frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6}\right] = 4\varepsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^{6}}{x^{7}}\right] = 0$

$$\frac{dU(x)}{dx} = 4\varepsilon\frac{d}{dx}\left[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^{6}\right] = 4\varepsilon\left[\frac{-12\sigma^{12}}{x^{13}} + \frac{6\sigma^{6}}{x^{7}}\right] = 0 \quad , \quad x = (2)^{\frac{1}{6}}\sigma$$

(b) Plug in numbers given, $x=2.95\times10^{-10}$ m

(c) The potential energy curve should look like the figure shown to the right.



(d) When $x= 4.5\times10^{-10}$ m, the two atoms are subject to a restoration form to bring them together to the equilibrium point ($x=2.95\times10^{-10}$ m)

(e) This can be proved by taking the first derivative of the potential, $\frac{dU}{dx}$>0, this is the force of the two atoms at that point, so it is a restoration force to bring them together.

4. **<u>Linear momentum conservation</u>: (15%)** Two masses are in a collision course with mass $\mathbf{M_1}$, $\mathbf{M_2}$ and velocities $\mathbf{V_1}$ and $\mathbf{V_2}$, respectively. Using Newton's third law, derive (or prove) the conservation of linear momentum in an isolated system.

Using Newton's 3rd law

$F_{12} = -F_{21}$ $\qquad m_1 \circ \xrightarrow{v_1} \quad \xleftarrow{} \circ \; v_2 \; m_2$

$F_{12} + F_{21} = 0$

$m_2 a_2 + m_1 a_1 = 0$

$$m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$

$$\frac{d(m_1 v_1)}{dt} + \frac{d}{dt}(m_2 v_2) = 0$$

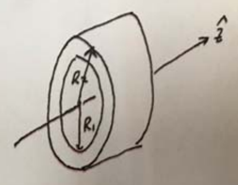
$$\Rightarrow \frac{d}{dt}(M_1 V_1 + M_2 V_2) = 0$$

$$M_1 V_1 + M_2 V_2 = \text{Constant}$$

— Conservation of linear Momentum

5. **<u>Moment of Inertia</u>: (20%)** What is the moment of inertia of a solid annular cylinder with total mass M, with outer radius R2, and Inner radius R1, rotating about any of its central axis?

(b) Annular cylinder



$$I = \int dm\, r^2 \qquad dm = 2\pi r H \rho\, dr$$

$$= \int 2\pi r H \rho\, dr\, r^2$$

$$= 2\pi H \rho \int_{R_1}^{R_2} r^3 dr = 2\pi H \rho \frac{1}{4}(R_2^4 - R_1^4)$$

But $M = \pi R_2^2 H\rho - \pi R_1^2 H \rho = \pi H \rho (R_2^2 - R_1^2)$

$$\therefore I = \frac{\pi}{2} H\rho (R_2^4 - R_1^4) = \frac{\pi}{2} H \rho (R_2^2 + R_1^2)(R_2^2 - R_1^2)$$

$$= \frac{1}{2} M (R_1^2 + R_2^2)$$

6. **<u>Solid sphere rolling down an inclined plan</u>: (20%)** A sphere rolls without sliding down an inclined plane making an angle θ with the horizon. The solid sphere has a total mass M, radius R and rotational moment of inertia I_{CM}; the inclined plane has a length L, and its height is H. (a) What is its kinetic energy when it is rolling down from top with angular velocity ω? (b) What is its translational velocity of the center of mass (V_{CM}) in terms of parameters given?

P10-10

$$a_{cm} = \frac{dv_{cm}}{dt} = R\frac{d\omega}{dt} = R\alpha$$

Check also Fig 10.28 (p 317)

The total kinetic energy of the rolling cylinder:

$$K = \frac{1}{2} I_p \omega^2$$

$I_p \equiv$ moment of inertia about point P.

$$= \frac{1}{2}(I_{cm} + MR^2)\omega^2$$

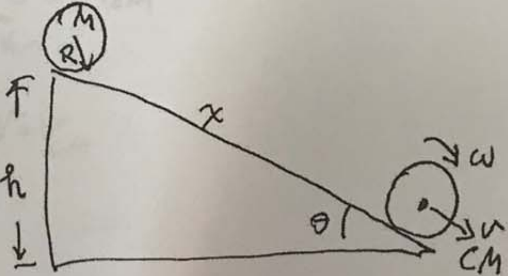
$$= \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} MR^2\omega^2$$

$$= \underbrace{\frac{1}{2} I_{cm}\omega^2}_{\text{Rotation about center of Mass}} + \underbrace{\frac{1}{2} M v_{cm}^2}_{\text{translation of center of mass}}$$

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2 \quad , \text{ But } v_{cm} = R\omega$$

$$= \frac{1}{2} I_{cm}\left(\frac{v_{cm}^2}{R^2}\right) + \frac{1}{2} M v_{cm}^2$$

$$= \frac{1}{2}\left(\frac{I_{cm}}{R^2} + M\right) v_{cm}^2$$



$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}\left(\frac{I_{cm}}{R^2} + M\right) v_{cm}^2 + 0 = 0 + Mgh$$

$$v_{cm} = \left(\frac{2gh}{1 + \frac{I_{cm}}{MR^2}}\right)^{\frac{1}{2}}$$