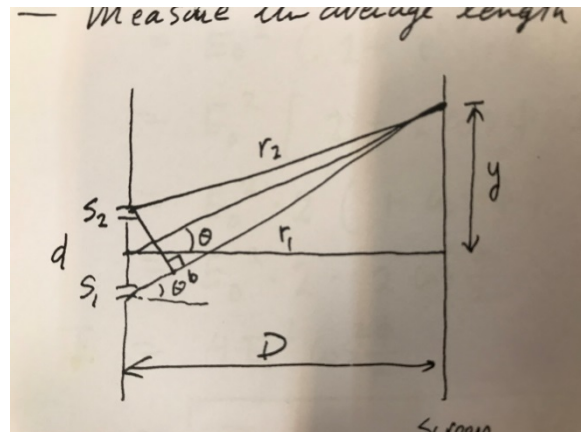


SN: _____, Name: _____

ABSOLUTELY NO CHEATING!

Problems (5 Problems, total 100%)

- Magnetism:** (20%) In a hydrogen atom, an electron of charge Q , mass m , moving around the nucleus with an angular velocity ω , period T , and radius r . (a) What is the electric current generated due to this electron moving in the atom? (b) What is the orbital angular momentum of the electron? (c) What is the magnetic momentum due to this motion? (d) Draw a schematic representation the motion of the electron.
- Oscillation in an LC circuit:** (20%) In an LC circuit, that is a circuit consist of a capacitor (C) and an inductor (L) connected in series. If the capacitor has a maximum capacity of charged to Q_{max} , (a)(5%) What is the total energy this circuit can store in this system when the current is I ? (b) (15%) What is the charge measured in the capacitor as a function of time, i.e. what is $Q(t)$?
- Double-slit interference:** (20%) Refer to the figure to the right; it is a general set up for a double-slit interference. In this case, if $D \gg d$, then r_1 can be considered parallel to r_2 . (a) Using the given parameter, derive the bright spots and dark spots that can be observed on the screen. (b) If we use a monochromatic light of wavelength $\lambda = 500$ nm, and the slit separation $d = 20$ μm , what is the angle θ measured from the horizon to the first bright spot?

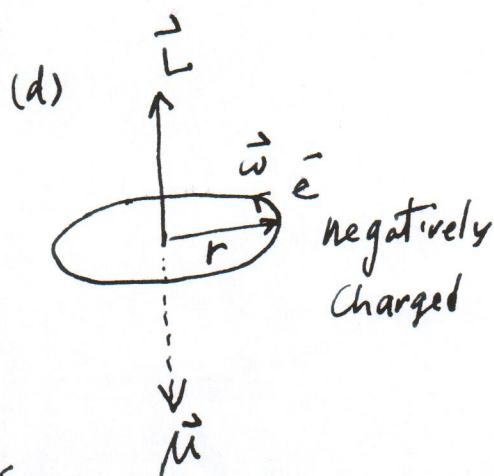


- Single slit diffraction:** (20%) A parallel beam of blue light (wavelength 420 nm) is incident on a small aperture. After passing through the aperture, the beam is no longer parallel but diverges at 1° to the incident direction. What is the diameter of the aperture? Note: for small angle θ , $\sin \theta \approx \theta$. (15%)
- Electric dipole in an electric field:** (20%) An electric dipole consist charges of $+q$ and $-q$ are separated by a distance $2a$ is placed in a uniform electric fields, making an angle θ between the dipole and the field direction. (a) What is torque generated due the electric force on the dipole? (b) If we rotate the dipole from an angle θ_1 to angle θ_2 , what potential energy we have changed?

1. for an electron moving around an atom as shown in the figure

$$(a) I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

Since electron is negatively charged. the current is in opposite direction as the electron moves.



$$\mu = IA = \frac{ev}{2\pi r} (\pi r^2)$$

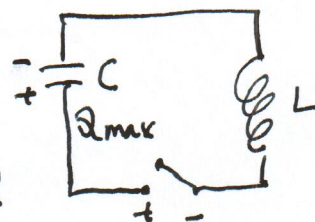
$$(b) L = \text{orbital angular momentum} \\ = m_e v^2 r$$

$$(c) \vec{\mu} = \frac{ev}{2\pi r} (\pi r^2) = \frac{1}{2} evr = \frac{m}{2m} evr \\ = \left(\frac{e}{2m}\right) \vec{L}, \text{ since } e \text{ is negative}$$

therefore, the directions of $\vec{L}$ and $\vec{\mu}$ are opposite to each other

2.

Refer to the figure to the right,



The total energy can be stored in this system is

(a) $U = U_C + U_L = \frac{1}{2C} Q^2 + \frac{1}{2} L I^2$

(b) Use the above relation

$$\begin{aligned} \frac{dU}{dt} &= \frac{d}{dt} \left(\frac{Q^2}{2C} + \frac{LI^2}{2} \right) = 0 \\ &= \frac{Q}{C} \frac{dQ}{dt} + LI \frac{dI}{dt} \\ &= \frac{Q}{C} \frac{dQ}{dt} + L \frac{dQ}{dt} \frac{d^2Q}{dt^2} \\ &= 0 \end{aligned}$$

Since the total energy of the system is constant, so the time derivative is zero

$$\rightarrow \frac{Q}{C} + L \frac{d^2Q}{dt^2} = 0$$

$$\rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{LC} Q = 0$$

Compare to

$$\frac{d^2x}{dt^2} = -\frac{k}{m} x = -\omega^2 x$$

$$\therefore \frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0$$

in mechanical system in a spring

$$\frac{d^2Q}{dt^2} + \omega^2 Q = 0$$

$$Q(t) = Q_{\max} \cos(\omega t + \phi)$$

ϕ can be determined by initial condition of the system

- 3 The lights going through S_1 and S_2 will
 (a) appear at the screen, but they travel at different paths. and the total path difference ΔL , according to the graph

$$\Delta L = d \sin \theta = m \lambda \quad \text{this will generate bright spots}$$

$$\Delta L = d \sin \theta = (m + \frac{1}{2}) \lambda, \quad \text{dark spots}$$

where θ is small $m = 0, 1, 2, \dots$

$$\Delta L = d \sin \theta = m \lambda$$

$$= d \sin \theta = (m + \frac{1}{2}) \lambda$$

- (b) If $\lambda = 200 \text{ nm}$, $d = 20 \mu\text{m} = 20000 \text{ nm}$
 first bright $m = 1$

$$d \sin \theta = 1 \cdot \lambda$$

$$\sin \theta = \frac{\lambda}{d} = \frac{200 \text{ nm}}{20000 \text{ nm}} = \frac{2}{200} = \frac{1}{100}$$

$$\therefore \theta = \sin^{-1}\left(\frac{1}{100}\right)$$

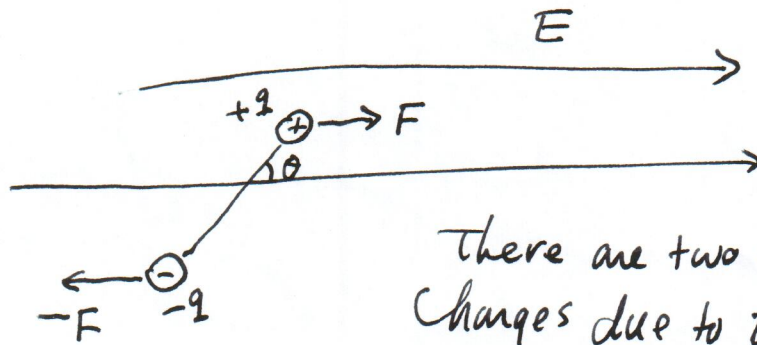
4. If d is the diameter of the aperture,

$d \sin \theta = 1 \lambda$ is the first minimum of the central bright diffraction pattern. $\theta = 1^\circ$

$$\sin \theta = \sin 1^\circ = \theta = 1^\circ = \frac{1}{180} \pi, \quad \pi = 3.14$$

$$\therefore d = \frac{\lambda}{\theta} = \frac{420 \times 10^{-9} \text{ m}}{3.14/180} = 24 \mu\text{m}$$

5



There are two forces on the two charges due to the electric force of the electric field.

$$P = \text{dipole} = 2aq =$$

$$F = \text{electric force} = q\vec{E}$$

(a) The torque $\tau = 2 \cdot qE \cdot a \sin\theta$

$$= PE \sin\theta$$

$$\therefore \vec{\tau} = \vec{P} \times \vec{E}$$

(b)

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} PE \sin\theta d\theta = PE \int_{\theta_i}^{\theta_f} \sin\theta d\theta$$

$$= PE (\cos\theta_i - \cos\theta_f)$$

$$= -PE (\cos\theta_f - \cos\theta_i)$$

$$= -\vec{P} \cdot \vec{E}$$